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Snow and Avalanche Physics: Physical Analysis of Factors Affecting the Stability of a Seasonal Continental Snowpack

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SNOW AND AVALANCHE PHYSICS

Physical Analysis of Factors
Affecting the Stability of a
Seasonal Continental Snowpack

James Frankenfield

A Report Submitted in Partial Fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

in Physics

at

Utah State University

Logan, UT

1989

Approved:

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I also appreciate the work of Steve Weiss, documents librarian. Steve went to great lengths to help me obtain many of the papers referenced in this paper. I would also like to express my appreciation to Duain Bowles of UDOT for his hospitality at Alta, his willingness to teach me about his job and his work, and for introducing me to many of the other experts in the field at Alta.

Jim Frankenfield

University of Maryland
European Division

January, 1990

Preface

Each section in this paper is intended to be as independent as possible. An attempt has been made to avoid references to other sections, and in particular to equations previously used. Hopefully the reader will not find it necessary to page back and forth very much. Figures and footnotes are numbered separately for each section.

A glossary has been included to assist the reader who is unfamiliar with some of the terms used. It includes entries of a qualitative nature (e.g. 'rime', 'graupel') as well as technical terms (e.g. 'Strouhal number').

The text was prepared using Wordperfect 4.1. Certain symbols were not available. When a standard symbol could not be used an attempt was made to use something as appropriate as possible. Thus σ is used for volume density rather than ρ , etc.

The study of snow is a fun and fascinating topic. Mountainous terrain offers a great natural laboratory, and fieldwork can be quite enjoyable (although sometimes a bit chilly). However, a rigorous study of problems in snow science also involves much science and mathematics. This paper will address, at an introductory level, some of the technical aspects of the study of snow and avalanches.

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I - Introduction

The study of snow cover is a discipline with a relatively short history. It is generally considered to have begun in 1931 in Switzerland with some federally sponsored studies which ultimately led to the formation of a Swiss National Institute for Snow and Avalanche Research in Davos. Prior to this there had been few organized studies, but some individual observations and informal studies had been published.¹ In the 1930's and 1940's the USSR and Japan initiated studies in this field. In the early 1950's the United States began studies of a military nature. These led to the formation of the Army Corps of Engineers Cold Regions Research and Engineering Lab (CRREL) in Hanover, New Hampshire.

In the forward to "Physics of Snow, Avalanches and Glaciers", a Soviet collection of papers published about 1970, the editors state:

"About forty years ago, the celebrated geophysicist B.P. Weinberg wrote that not a single property of snow was known in spite of the colossal amount of snow covering the terrestrial surface. As a result of the work carried out in the last ten years, studies on snow cover have come to be recognized as an independent discipline earning for itself a place among other sciences."²

¹ "Snow Structures and Ski Fields", (15); This is still considered a classic source of field observations.

² "Physics of Snow, Avalanches and Glaciers", (2), p. i

The study of snow was born of several motivating factors. The earliest studies in Switzerland and Japan were primarily due to concern over the hazards presented by avalanches. Avalanches in the Alps tend to be larger and more destructive than those typical of the mountains in the United States. The Alps also contain a large number of small settlements, exposing more people and more transportation corridors to a higher risk than in other parts of the world. In Japan, many of the studies were done by the railroad companies. A large number of studies conducted by the United States and the USSR grew from a military interest in the polar regions.

Another source of interest in seasonal snow cover has been hydrology. In certain regions, such as the western United States, water is of great importance. Hydrological models and forecasts have been, and continue to be, a major source of interest in snow science.

In the United States, about 10,000 avalanches per year are reported. Unlike Europe, most regions prone to avalanches in this country are sparsely inhabited. The risks are primarily associated with recreation and transportation. Regions of high risk include the San Juan mountains in Colorado, the Wasatch range in Utah, and the Teton range in Wyoming. However, avalanches occur often throughout the western part of the country.

The study of snowcover begins with the study of snow formation. The type of snow which falls can vary widely and has many implications

for the stability of the snowpack on the ground. There is a great deal of interesting work related to the formation of snow crystals. Once snow is on the ground, it undergoes constant change. The processes are primarily thermodynamic in nature and depend greatly on temperature gradient across the snowpack. Albedo and other radiation considerations play an important role. At any given point in time, one can attempt to analyze the stresses and strains within the snowpack. Finite element analysis is one method which has been applied to this task.

The most common method of hazard control is through the use of explosives. An analysis of this requires some knowledge of shock waves. Most of the work in this area has been by the military and has been primarily concerned with polar snow cover.

While much has been learned in the last forty to sixty years, a thorough understanding of snowcover remains an elusive and open ended goal. Snowcover, particularly seasonal cover, is highly anisotropic and is nonhomogeneous both spatially and temporally. Clearly, many questions remain unanswered (and perhaps unasked).

II - Formation and Deposition of Snow

Snow can take many different forms. The original form of the snow, as deposited, affects the rate at which it changes when it becomes part of the snowpack. Although most snow is formed in the atmosphere and falls as precipitation, crystals can form directly at the point of deposition. Crystals formed on the snow surface in a manner analogous to dew are called 'surface hoar'. Rime is also formed at the point of deposition.

The original form of the crystals also has a direct impact on the stability of the snowpack. Graupel sometimes creates a persistent weak layer, while surface hoar usually does. New snow consisting of interlocking dendritic structures can create a cohesive, and relatively strong, layer. An important consideration is how well a layer adheres to the existing snow surface when it is deposited.

The terms 'ice crystal', 'snow crystal', 'snowflake' and 'grain' have particular meanings and are not interchangeable. The reader who is unfamiliar with these terms should refer to the glossary at this point.

In this section, the general conditions under which snow forms are reviewed. Some models for ice nucleation and crystal growth are discussed. Finally, the most common classifications for types of snow crystals and grains are introduced.

A - Meteorological Factors

When a mass of air moves upward and cools adiabatically it reaches a temperature called the dew point, at which water droplets and clouds form. If this mass continues to rise, additional cooling and condensation result in precipitation. For most purposes, non-adiabatic effects are small enough to be ignored in vertical air flow. Since air is a poor thermal conductor, a particular mass of air tends to retain its own thermal identity and does not usually mix well with surrounding air masses. Near the earth's surface most processes are not adiabatic due to turbulence, friction and other ground effects.

Vertical Motion of Air Masses

There are four types of vertical motion which can result in significant amounts of precipitation. They are: horizontal convergence, orographic lifting, convective lifting, and frontal lifting.

Horizontal convergence results when the winds direct air into one area, at which point it must rise. At some upper level, it then diverges. The most common source of this type of lifting is a low pressure system. For this reason it is often referred to as cyclonic lifting. (See Fig. 1). However, it can also result from surface friction effects.© The resulting vertical speed is

© "Atmosphere, Weather & Climate", (13), p.94, p.109

on the order of 0.05 - 0.1 m/s.

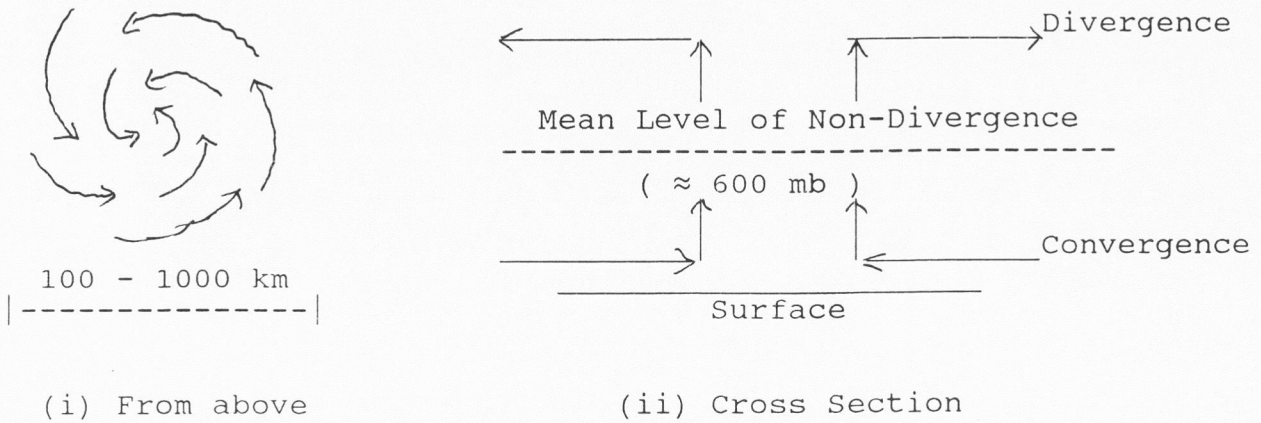


Fig. 1 - Cyclonic Lifting

Orographic lift is caused when air flows against a terrain barrier and is a powerful mechanism. Vertical speeds vary from .05 up to 2 m/s. The vertical velocity will depend on the slope of the terrain barrier and the angle of incidence of the horizontal air flow. (See Fig. 2).

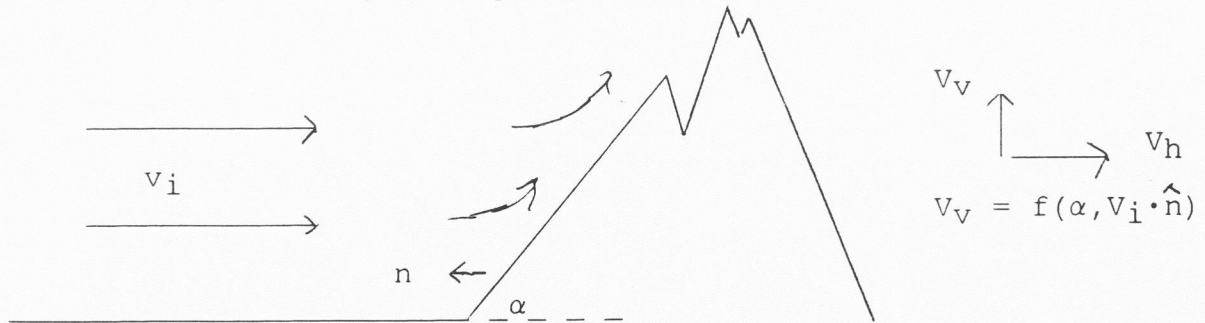


Fig. 2 - Orographic Lifting

Convective lifting results from differential heating of air near the ground. Localized masses of buoyant air result. Under the right conditions, these can rise at great velocities, ranging from 1 to 10 m/s. The resulting precipitation is usually very localized and often heavy. This mechanism is of great importance

in summer, when strong thunderstorms result. In winter it is a relatively minor factor in most locations. However, in certain areas such as the snowbelts south and east of the great lakes air flows over a warmer body of water and convection is important.

A 'front'[⊕] is the area separating two large masses of air. Differences in the density and motion of these distinct masses usually results in lifting. Vertical windspeeds vary between .05 and .2 m/s. In a cold front, invading cold air causes rapid lifting of the warm air being displaced. The interface has a steep slope and rapid lifting results. In a warm front, warm air is forced to rise above existing cold air. The interface in a warm front is less steep, resulting in a lower rate of lifting and a lower rate of advance of the front. (See Fig. 3). Frontal systems result in widespread precipitation. The greatest amount of activity occurs in the northeast quadrant of a surface low pressure area, where frontal and cyclonic lifting combine.

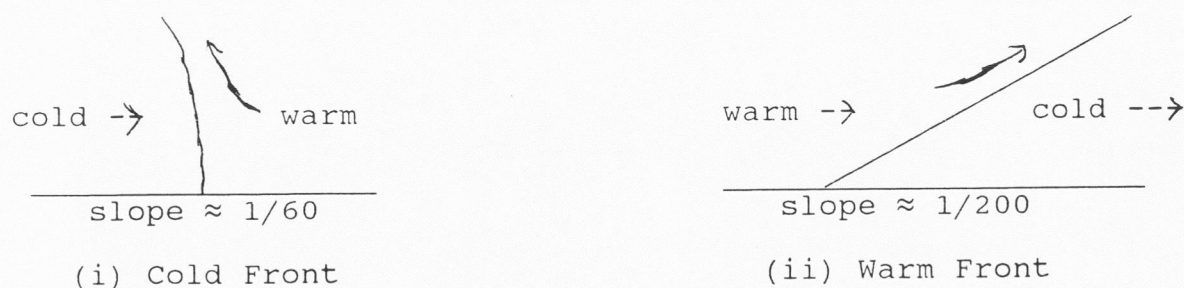


Fig. 3 - Frontal Lifting

⊕ The term 'front' was proposed during WW I by a group of meteorologists in Norway to describe regions of turbulent mixing where differing air masses meet. (13), p. 163.

Redistribution of Snow by Wind

The local depth of the snowcover is related to many factors such as elevation, slope and aspect, vegetative cover, and radiation balance. This section will serve as an introduction to the nonuniform distribution of snow resulting from the effect of winds over terrain features. This is a particularly important factor in analyzing the stability of the snow. Wind distribution and redistribution result in pockets of heavy, packed snow. These 'wind slabs' are generally not stable, and often lead to avalanches.♥

At altitudes greater than about one mile, air motion is directed in a manner such that the pressure gradient forces are in equilibrium with the Coriolis force. Winds at these altitudes are called "geostrophic winds" (See Fig. 4).



Fig. 4 - Geostrophic Wind in Northern Hemisphere

♥ For a discussion of other effects resulting in non-uniform snow depth see: "Handbook of Snow";(11); Ch. 5

The velocity of the geostrophic wind is given by:

$$V_g = (F_C \sigma)^{-1} \cdot dp/dn ; F_C = -2\Omega \sin \phi$$

where: F_C = Coriolis parameter

σ = density of air

Ω = angular velocity ($2\pi/24$ rad/hr for Earth)

ϕ = latitude

dp/dn = pressure gradient

At the earth's surface, the boundary condition for fluid flow requires zero air speed. Therefore, the wind velocity increases to its geostrophic value across a boundary layer. Over rough terrain the boundary layer is thicker and the wind speed changes slower than over flat terrain. The Coriolis force decreases with decreasing velocity, so the actual flow which results is in the form of a spiral. From Fig. 4, one can see that the air will spiral inward for a low pressure system, leading to cyclonic lifting. (See last section, Fig.1).

Since the boundary layer exhibits velocity gradients, shear stresses occur in the wind flow near the earth's surface. This shear is highest at the surface, and is effected by surface features such as ridges (See Fig.5). This moves loose snow, resulting in a redistribution.

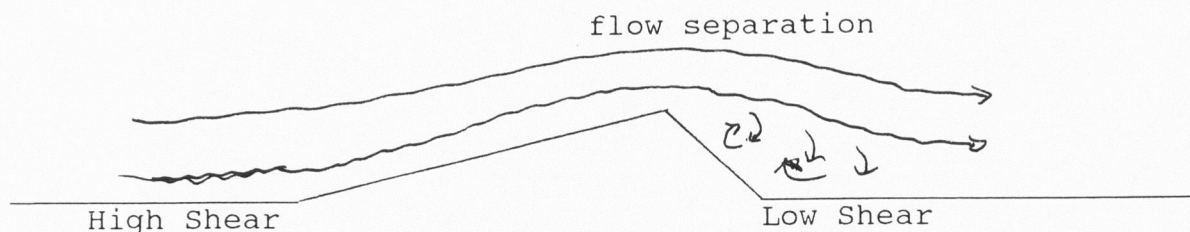


Fig. 5 - Wind Flow Over a Ridge

This redistribution of snow is due to three transport mechanisms: rolling, saltation, and turbulent diffusion (See Fig. 6). Rolling occurs on the snow surface and does not contribute significantly to drifting or to the loading of avalanche slopes. Saltation involves the motion of particulates just above the surface. A particle is picked up due to vertical motion in eddy currents or by aerodynamic lift. It is accelerated horizontally by an aerodynamic drag force while being returned to the surface by gravity. It is believed to be important in the first 10 - 30 cm above the surface. Turbulent diffusion occurs when particles are suspended in the wind, with the lighter ones rising higher. Most of the mass transport due to this mechanism occurs in the first one meter or so above the surface.

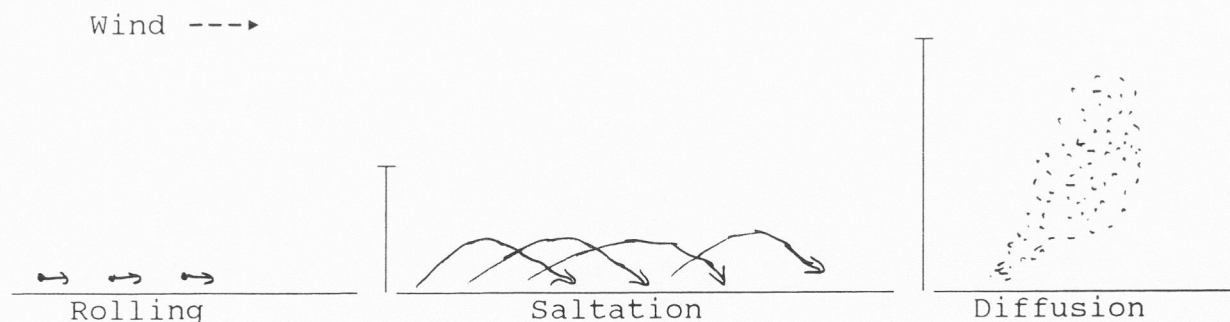


Fig. 6 - Transport Mechanisms

It is not clear whether saltation or turbulent diffusion are equally important or whether one usually predominates. Theories have been developed for both.♦ Whatever the combination, the resulting mass transport can be very rapid. Deposition rates in

♦ "Dynamics of Snow and Ice Masses";(1); Ch. 6, Sec 4

an area of wind loading at Berthoud Pass, Colo., have been observed to reach 45 cm/hr.♣

As Fig. 5 shows, snow is moved from one side of a terrain barrier to the other. In mountainous areas this results in localized areas of heavy snow deposition. The blowing snow particles are usually pulverized to one tenth their original size, resulting in a higher density in the deposition area. Because of the smaller particle size, more contacts between particles result and these wind depositions firm up quickly.♠ For this reason, they are generally referred to as "wind slabs" and present a potential hazard.

At sharp terrain bends, snow accumulates in the form of hard overhanging structures called cornices. These can grow to large sizes and can initiate an avalanche if they fall onto the windslabs below.

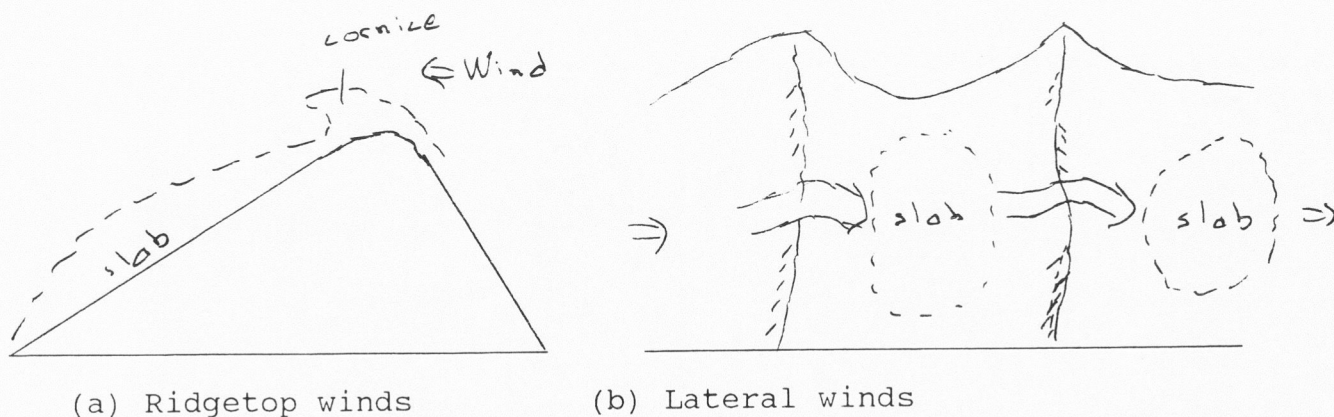


Fig. 7 - Windslab Formation

♣ "Avalanche Handbook"; (12); Ch. 2

♠ See section on Equi-temperature Metamorphism

B - Crystal Growth and Classification

For the formation of snow to occur, the ambient temperature must be below freezing and supercooled water must be present. Nuclei must also be available to initiate the process of ice nucleation. The habit (orientation) of an ice crystal depends on the environmental conditions during nucleation. When snow crystals reach the ground they may have widely varying characteristics depending on how they were nucleated and on the conditions encountered on their way down. In order for those interested in snow to communicate in a meaningful fashion, good classification schemes for these snow crystals are required.

Ice Nucleation

The initiation of the ice phase from the vapor phase occurs via the nucleation of a small ice embryo. If this occurs on a foreign particle, it is referred to as "heterogeneous" nucleation. If such particles do not play a role and water molecules combine directly it is called "homogeneous" nucleation. The chance combination of water molecules required for homogeneous nucleation generally begins to occur at temperatures below -40°C . For this reason, and since the atmosphere generally contains abundant nuclei, we will be concerned primarily with heterogeneous nucleation.

At any given time, one cubic centimeter of air in the lower atmosphere contains hundreds to thousands of particles, most ranging in size from 0.01 to 1.0 μm . Major sources of nuclei, in order of significance, are: dust, industrial plants and forest fires, particles of extraterrestrial origin, and organic matter from vegetation. These nuclei have a wide variety of chemical compositions and are characterized by different activation temperatures. Those active between -5 and -10 $^{\circ}\text{C}$ are considered efficient, those active below -20 $^{\circ}\text{C}$ are poor. The concentration of 'active' nuclei increases rapidly with decreasing air temperature. Between -10 $^{\circ}\text{C}$ and -30 $^{\circ}\text{C}$ it increases by about a factor of ten with each 4°C decrease.

The concentration of particles can be increased locally either naturally or artificially. One reason the Wasatch mountains in Utah receive large amounts of snow is that most storms cross the Great Salt Lake and pick up not only moisture, but also nuclei. Clouds can also be seeded artificially from aircraft. Silver Iodide has been found to be effective in cloud seeding.

Heterogeneous nucleation actually consists of two distinct processes: direct deposition and the freezing of supercooled droplets. Thus we distinguish between deposition nuclei and freezing nuclei. The solubility of the nucleus is an important

• "Handbook of Snow"; (11); Ch. 4

factor and helps determine its suitability for deposition vs. condensation. Since the saturation vapor pressure is less over a solution droplet than over a comparable pure water droplet, condensation can begin on soluble nuclei before the air reaches saturation. For sodium chloride nuclei it can begin at 78% relative humidity. On the other hand, condensation will not occur on an insoluble particle in air which is at its saturation point. With small supercooling and slight supersaturation, condensation should be more rapid than deposition. ■

Now we consider the rate of nucleation. Assume that a collection of N droplets all have the same particulate content and are all at the same temperature so that they all have the same probability of freezing at any instant of time. This leads to a statistical, or stochastic, model. The probability of nucleation occurring in volume V in the time interval 0 to t is:

$$P(V,t) = VJ_F t$$

where the nucleation rate J_F is the number of ice embryos which can form spontaneously from supercooled water in a unit volume per unit time. Expressions for J_F have been derived from lengthy and detailed thermodynamic considerations. ○ If N_t is the number of drops frozen at time t , then

$$P(V,t) = N_t/N$$

and

$$N_{t+dt} = N_t + (N - N_t)VJ_F(T)dt .$$

■ "Ice Physics"; (28); Sec. 7.3.6

○ Ibid.; Sec. 7.3

Dividing this by N gives

$$P(V, t+dt) = P(V, t) + [1-P(V, t)]VJ_F(T)dt ,$$

or

$$P(V, t+dt) - P(V, t) = [1-P(V, t)]VJ_F(T)dt = \frac{d}{dt} P(V, t)dt ,$$

so that

$$[1-P(V, t)]^{-1} dP(V, t)/dt = VJ_F(T) ,$$

and

$$\ln [1-P(V, t)] = - \int_0^t VJ_F(T)dt ,$$

where temperature T is generally a function of time.

Assuming $P(V, t) \ll 1$ and using the series representation of the logarithm,

$$\ln (1+x) = x - x^2/2 + x^3/3 \dots , \quad x \in (-1, 1] ,$$

we obtain

$$P(V, t) = V \int_0^t J_F dt .$$

If the drops are cooled at a constant rate $\beta = dT/dt$, and $T=0^\circ\text{C}$ at $t=0$

$$P(V, t) = (V/\beta) \int_0^{T_S} J_F dT ,$$

where T_S is the supercooling, $T_S = |T|$ where $T < 0^\circ\text{C}$.

Expressions for J_F indicate that the integral exhibits exponential behavior,

$$\int_0^{T_S} J_F dT = D \exp (T_S/\Gamma_0) ,$$

where D and Γ_0 are constants.

Our stochastic model now results in

$$P(V, t) = (DV/\beta) \exp (T_S/\Gamma_0)$$

and predicts that freezing events occur more frequently as temperature decreases. The change in probability per degree drop in temperature is:

$$dP(V,t)/dT_S = (DV/\beta\Gamma_0) \exp (T_S/\Gamma_0) .$$

A second model for the rate of nucleation can be developed which assumes every drop nucleates at a temperature determined by the most effective nucleus it contains. This is referred to as a singular model. In this model no freezing events occur at a fixed temperature. The concentration $n(T_S)$ of ice nuclei which become effective between 0 and $-T_C^0C$ can be modelled exponentially:

$$n(T_S) = n_0 \exp (T_S/\Gamma_0) .$$

If the distribution of ice nuclei contained in a drop is random, then the probability of it freezing while being cooled to $-T_S^0C$ is given by Poisson statistics as:

$$P(T_S) = 1 - N/N_0 = 1 - \exp[-n(T_S)/V].$$

If $P(T_S) \ll 1$ we can use a series approximation for the exponential:

$$1 - e^x = 1 - [1 + x + x^2/2 + \dots]$$

so that:

$$P(T_S) \approx n(T_S)V = n_0V \exp (T_S/\Gamma_0).$$

Now we can calculate the probability of freezing per degree drop in temperature and compare with the stochastic model:

$$dP(T_S)/dT_S = (n_0V/\Gamma_0) \exp (T_S/\Gamma_0) .$$

Both models predict an exponential rise in the probability of freezing with decreasing temperature. However, in the singular model no freezing occurs at a constant temperature, which is not true in the stochastic model. Also, in the stochastic model the probability of freezing per degree drop in temperature depends on the rate of cooling (β), while β does not appear in the singular model.

Experimental data do not conform to either model in general, but in some cases appear to approach one or the other as limiting cases. The nucleation temperature seems to be determined mainly by local properties and follows the singular model more closely than the statistical. Growth of ice embryos to critical size tends to exhibit some amount of random fluctuation, and the statistical model plays a major role.©

To show that the probability of freezing for a droplet is determined primarily by the distribution of characteristic temperatures of the nuclei it contains, we write the probability of freezing for a drop as

$$P(T,t) = \int P_1(T,T_C) P_2(T_C,t) dT_C ,$$

where: T = Temperature

t = time

© For a discussion of experimental studies, see "Ice Physics";(28); Sec. 7.4

T_C = Characteristic temperature for nuclei in drop

P_1 = Probability of the nuclei becoming active within a unit time interval at temperature T

P_2 = Number of nuclei which have not become active at time t .

The probability function P_1 varies rapidly with temperature, with nucleation generally occurring within 0.25°C of T_C . Therefore, we can represent $P_1(T, T_C)$ by a delta function

$$P_1(T, T_C) = \pi_i \delta(T_i - T_C)$$

where π_i indicates a product over the characteristic temperature of all nuclei in the drop. Now

$$\begin{aligned} P(T, t) &= f \pi_i \delta(T_i - T_C) P_2(T_C, t) dT_C \\ &= \pi_i P_2(T_i, t) . \end{aligned}$$

This shows that the probability of freezing is determined only by the distribution of characteristic temperatures for the nuclei.

Crystal Growth and Snowflake Formation

After nucleation occurs, the ice crystal will grow due to a difference in vapor pressure. It quickly reaches a size at which it develops a fall velocity relative to the cloud droplets. As it falls it sweeps up additional cloud droplets and grows by a riming process.

Initial growth after nucleation is described by the Bergeron-Findeisen theory. This theory is based on the fact that the relative humidity of air is greater with respect to an ice surface than with respect to a water surface. Conditions in a cloud may represent only slight supersaturation with respect to water but 10 - 20% supersaturation with respect to ice. At temperatures between -5 and -25°C the difference in saturation vapor pressure can exceed 0.2 mb. The result is that supercooled water droplets evaporate and vapor molecules are directly deposited on the ice crystals. Some successful cloud seeding experiments have been based on the Bergeron theory, adding support to it.

When the crystal diameter is less than a few hundred microns droplet capture is negligible and the time rate of mass increase is given by^{⊙⊙}

$$dm/dt = 4\pi CDFA_C(\sigma_{\infty} - \sigma_0)$$

where: C = shape factor
 D = diffusivity of water vapor in air
 F = ventilation factor
 A_C = accommodation coefficient
 σ_{∞} = vapor density away from crystal
 σ_0 = vapor density at crystal surface .

The ventilation factor F depends on motion relative to air and the accommodation coefficient A_C accounts for temperature and surface effects.

⊙⊙ "Handbook of Snow"; (11); Ch. 4

Once a crystal begins to fall it will grow by accretion (riming) if cloud droplets collide with it and adhere to the surface. The ratio of the number of droplets which actually impact on the crystal to the number swept out by it inside a column of air enclosed by the crystals cross sectional area and fall distance is called the collision efficiency. The collision efficiency of different crystal types is still poorly defined despite much experimental and theoretical work on the topic.

The adhesion efficiency is defined as the ratio of the number of droplets that freeze onto the crystal to the total number of contacts. It is very close to one and is often ignored in calculations concerning riming.

The mass growth rate due to riming of a crystal of approximately circular cross section falling through a supercooled cloud is[⊙]

$$dm/dt = \pi r^2 abwW$$

where: r = radius of crystal
 a = adhesion efficiency
 b = collision efficiency
 w = fall velocity relative to droplets
 W = water content of cloud .

Estimates based on this equation predict that a crystal of radius 250 μm may grow to a 1 - 2 mm pellet of graupel in 10 to 20 min. Hail is one possible result of this riming process.

⊙ Ibid.

Crystal Orientation (Habit)

Ice has a hexagonal crystal structure. Thus there is a crystallographic axis oriented from the bottom to the top of a hexagonal prism, and three axes 60° apart in the basal hexagon. The first is called the c-axis and the latter three are called the a-axes. (See Fig. 8). The habit of a crystal formed in the atmosphere is determined by the relative rates of growth along these axes.

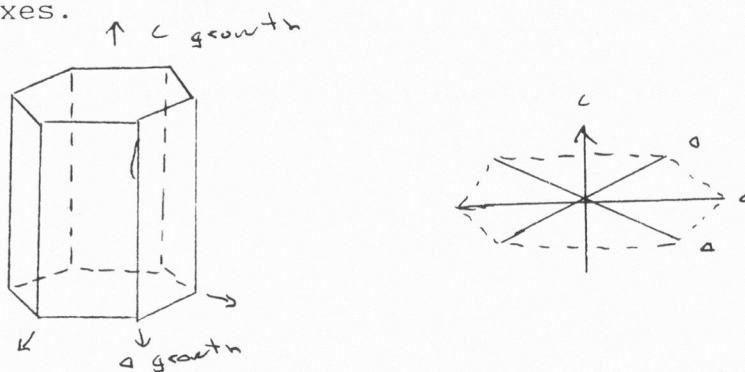


Fig. 8 - Crystal Structure of Ice

Taking c to be the length of a crystal parallel to the c-axis and a to be the hexagonal diameter of the base, two basic habits are defined: prism-like ($c/a > 1$) and plate-like ($c/a < 1$). It has been firmly established by numerous experiments that the basic habit of an ice crystal is determined by the temperature at which it forms. Between 0 and -4°C and also between -9 and -22°C plate-like crystals form. In the range of -4 to -9°C and below -22°C prism-like crystals form. The transition temperatures reported by different researchers vary by one degree or so, but the transitions are abrupt.

The linear growth rates as a function of temperature for the c direction and a directions have been measured and are shown in Fig. 9. They cross at -5.3°C and -9.5°C , in general agreement with the transition temperatures of -4°C and -9°C .

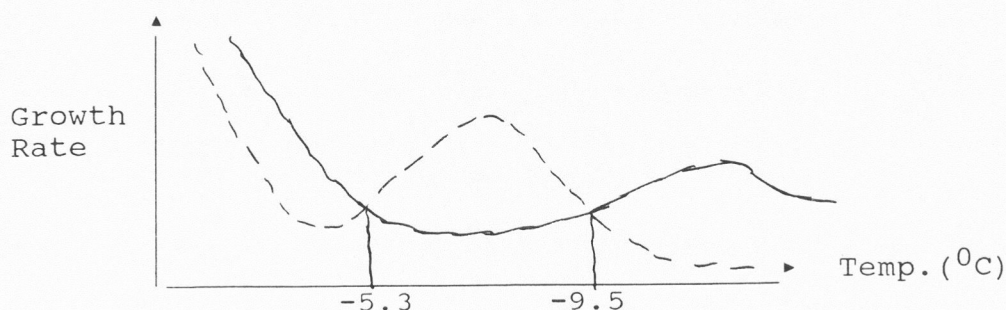


Fig. 9 - Linear Growth Rates^{⊙♥}

If the temperature varies during growth, the habit will change accordingly. In one experiment, needles were grown between -3.5°C and -5.5°C . The temperature was then changed to -1.5°C and plates formed on the ends of the needles. These plates did not thicken much once they were established.^{⊙♦} When crystals form in a cloud they may be exposed to different temperatures as they fall through a cloud or clouds, resulting in such hybrids.

The secondary growth features are determined by the level of supersaturation. The effect of increasing supersaturation on plate-like crystals is: very thick plates ▶ thick plates ▶ sector plates ▶ dendrites. The effect on prism-like crystals is: solid prism ▶ hollow prism ▶ needle.

^{⊙♥} based on "Growth Rates and Habits of Ice Crystals Grown from the Vapour Phase"; (43)

^{⊙♦} "Ice Crystals Grown from the Vapour"; (38)

Classification of Snow Crystals

In 1951 the International Commission on Snow and Ice proposed a classification scheme for solid precipitation. It was a simple scheme based on the major types of snow crystals plus graupel and hail. The categories were: plates (F1), stellar crystals (F2), columns (F3), needles (F4), spatial dendrites (F5), capped columns (F6), irregular (F7), graupel (F8), pellets (F9), and hail (F0). Category F7, irregular, was like a 'miscellaneous' file and too much ended up in it. A more detailed scheme was necessary. The ICSI system has fallen into disuse, although it may be encountered when reviewing older literature.

A better scheme was proposed by Mogono and Lee in 1966. It will accommodate most observed crystals, and is the system most widely used today. It has 80 categories, including many hybrid types. It is reproduced here as Fig. 10.

Fig. 10 - Mogono/Lee Classification Scheme

	P6b Plate with spatial dendrites		CP3d Plate with scrolls at ends		R3c Graupel-like snow with nonrimed extensions
	P6c Stellar crystal with spatial plates		S1 Side planes		R4a Hexagonal graupel
	P6d Stellar crystal with spatial dendrites		S2 Scalelike side planes		R4b Lump graupel
	P7a Radiating assemblage of plates		S3 Combination of side planes, bullets, and columns		R4c Conelike graupel
	P7b Radiating assemblage of dendrites		R1a Rimed needle crystal		I1 Ice particle
	CP1a Column with plates		R1b Rimed columnar crystal		I2 Rimed particle
	CP1b Column with dendrites		R1c Rimed plate or sector		I3a Broken branch
	CP1c Multiple capped column		R1d Rimed stellar crystal		I3b Rimed broken branch
	CP2a Bullet with plates		R2a Densely rimed plate or sector		I4 Miscellaneous
	CP2b Bullet with dendrites		R2b Densely rimed stellar crystal		G1 Minute column
	CP3a Stellar crystal with needles		R2c Stellar crystal with rimed spatial branches		G2 Germ of skeleton form
	CP3b Stellar crystal with columns		R3a Graupel-like snow of hexagonal type		G3 Minute hexagonal plate
	CP3c Stellar crystal with scrolls at ends		R3b Graupel-like snow of lump type		G4 Minute stellar crystal
					G5 Minute assemblage of plates
					G6 Irregular germ

	N1a Elementary needle		C1f Hollow column		P2b Stellar crystal with sectorlike ends
	N1b Bundle of elementary needles		C1g Solid thick plate		P2c Dendritic crystal with plates at ends
	N1c Elementary sheath		C1h Thick plate of skeleton form		P2d Dendritic crystal with sectorlike ends
	N1d Bundle of elementary sheaths		C1i Scroll		P2e Plate with simple extensions
	N1e Long solid column		C2a Combination of bullets		P2f Plate with sectorlike extensions
	N2a Combination of needles		C2b Combination of columns		P2g Plate with dendritic extensions
	N2b Combination of sheaths		P1a Hexagonal plate		P3a Two-branched crystal
	N2c Combination of long solid columns		P1b Crystal with sectorlike branches		P3b Three-branched crystal
	C1a Pyramid		P1c Crystal with broad branches		P3c Four-branched crystal
	C1b Cup		P1d Stellar crystal		P4a Broad branch crystal with 12 branches
	C1c Solid bullet		P1e Ordinary dendritic crystal		P4b Dendritic crystal with 12 branches
	C1d Hollow bullet		P1f Fernlike crystal		P5 Malformed crystal
	C1e Solid column		P2a Stellar crystal with plates at ends		P6a Plate with spatial plates

III - The Seasonal Snowpack

In this section, the structure of the seasonal snowpack, and the instabilities which can occur within it, are discussed. The goal is to develop a basic understanding of how conditions conducive to avalanches arise and of the mechanisms by which slides are initiated.

Two general types of avalanches occur. Loose snow avalanches generally occur during or shortly after a storm. Fresh, soft, unconsolidated snow slides down off of slopes which are too steep to allow it to collect and settle (typically greater than about 45 degrees). While these slides can be large on occasion they are rarely responsible for injuries or damage.

Slab avalanches usually occur after the snow has begun to consolidate. Generally a large, cohesive layer of snow (a slab) slides on a layer of weak, incohesive crystals. Typical weak layers are temperature-gradient (TG) crystals and buried layers of surface hoar. Slab releases are extremely destructive, and are often triggered by some external source such as a skier. They occur most frequently on slopes with inclines of 25-50 degrees.

A - Metamorphism

After snow is deposited and becomes part of the snowpack the crystals undergo changes. These processes of change are referred to as metamorphism. Thermodynamically, an equilibrium state is approached. The processes can be examined through free energy considerations and by considering the transport of heat and water molecules through the snowpack.

Two mechanisms are responsible for most metamorphism, each being driven by different thermodynamic conditions. When all or part of the snowpack is at essentially the same temperature, "equitemperature" metamorphism (ET) dominates. This process results in the breakup of any dendritic assemblies, and eventually in the sintering together of adjacent grains. Since it promotes destruction of the original crystal structure, it is sometimes referred to as a form of "destructive metamorphism". This mechanism increases the strength of the snowpack.

When part of the snowpack is exposed to a steep temperature gradient, "temperature-gradient" metamorphism (TG) is the dominating process. Since vapor pressure is proportional to temperature, water vapor is transported from warmer parts of the snowpack to the colder regions where it is deposited as ice. TG metamorphism promotes the growth of individual faceted crystals. These crystals can grow quite large and do not sinter together. The result is a layer which is weak and incohesive. Such layers

often fail under shear loading and are thus a major cause of avalanches. This process is referred to as a type of "constructive" metamorphism.

A third mechanism called "melt-freeze" metamorphism occurs when free water is introduced into the snowpack, generally through rain or melting at the surface. The effect on the snowpack is predictable. During the freeze part of the cycle very high strength exists and the snowcover is stable. During the melt part of the cycle, strength decreases drastically and wet, slow slides occur. These effects are seen regularly in spring. This process will not be discussed further here.⊙

Metamorphism plays a major role in the properties of snow. Many thermal, mechanical, and electromagnetic properties have been studied and tabulated.⊙

Equitemperature Metamorphism

The snow grains within a snowpack will seek an equilibrium state, which can be defined as the state with minimum free energy. New snow has many dendrites, and thus has very high surface energy per unit mass. The first sign of equitemperature metamorphism is the rounding of sharp corners. Next the snow will tend to form

1 see: Colbeck, S.; 'Theory of Metamorphism of Wet Snow';
US Army CRREL Research Report 313; 1973

⊙ see, for instance, "Handbook of Snow", (11), Ch. 7

spherical grains. Finally, the larger grains will grow at the expense of the smaller ones, and 'necks' will form between grains (i.e. 'sintering' occurs).

The effects of surface area can be examined through basic thermodynamic considerations. We begin with the First Law of Thermodynamics, or Conservation of Energy

$$dU = \delta Q - \delta W$$

where dU = change in energy

δQ = heat added

δW = work done .

If we assume that work done is manifest only as changes in volume and surface area, then

$$\delta W = PdV + \alpha dA$$

where P = pressure

V = volume

α = surface energy/unit area

A = area .

Now we make use of the Second Law of Thermodynamics

$$dS \geq \delta Q/T$$

where S = entropy

T = temperature at point of heat transfer .

Since addition or removal of heat is occurring at a constant temperature, we can consider this to be a reversible process and use the equality.

We can now rewrite the First Law as

$$dU - TdS + PdV + \alpha dA = 0 .$$

If we assume the pressure is constant in the vicinity of the grain of ice, and that α is constant, then

$$dT = dP = d\alpha = 0$$

and we can again rewrite the First Law,

$$dU - TdS - SdT + VdP + PdV + \alpha dA + Ad\alpha = 0 ,$$

or $dG = 0$

where $G = U - TS + PV + \alpha A =$ Gibbs Free Energy .

Now we recall that $dT = dP = d\alpha = 0$ and assume that $\delta Q \ll T$ so that $dS \approx 0$ and

$$dG \approx dU + PdV + \alpha dA$$

or

$$dG/dV \approx dU/dV + P + \alpha dA/dV .$$

Since dU/dV and P are constant, dG/dV will decrease only if dA/dV decreases. Thus a decrease in the ratio of surface area to volume is equivalent to a decrease in free energy.

It has been shown that the primary mechanism by which these changes occur is via transport of water molecules in the vapor phase. Volume diffusion and surface diffusion occur at a rate which is about four decades slower.♥ At very high densities, about 580 kg/m^3 , volume diffusion becomes important.♦ Water vapor transport is due primarily to differences in curvature of

♥ "The Sintering and Adhesion of Ice", (25)

♦ "The Role of Volume Diffusion ...", (26)

the grains.

To describe the flux of vapor between two grains, we begin with Fick's Law

$$j = -D \nabla C$$

where j = flux of vapor

D = diffusion coefficient

C = concentration of vapor .

In one dimension this becomes

$$j = -D(C_1 - C_2) / \Delta x$$

for the flux from grain two to grain one (Fig. 1).

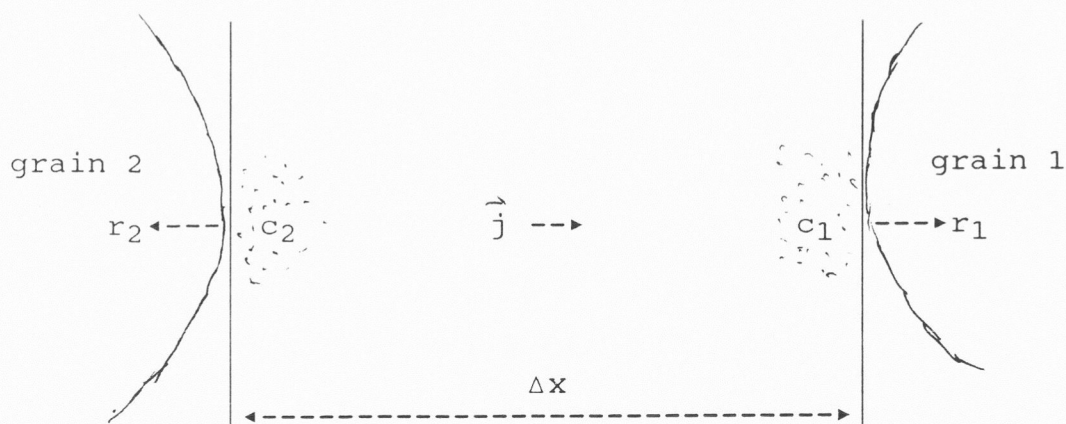


Fig. 1 - Flux Between Grains

Water vapor in air at atmospheric pressure obeys the ideal gas law, so

$$C = P/RT \quad R(\text{H}_2\text{O}) = .462 \text{ kJ/kg K} .$$

Now Fick's Law becomes

$$j = -D/R\Delta x(P_1/T_1 - P_2/T_2) .$$

Next we make use of Kelvins Equation for vapor pressure over a curved surface,

$$P = P_0 \exp(\alpha\delta^3/kTr) \approx P_0 (1 + \alpha\delta^3/kTr)$$

where P_0 = saturation vapor pressure over a plane surface

α = mean intermolecular distance

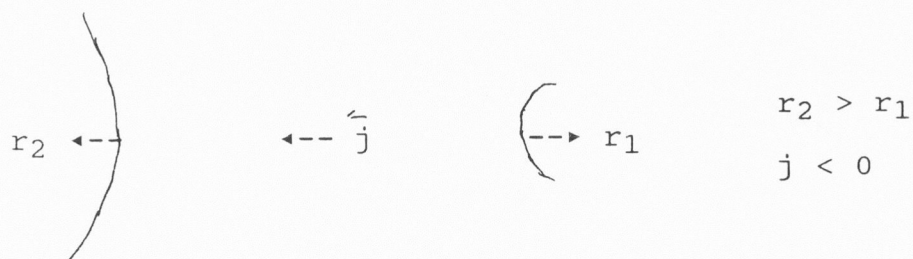
r = distance from center of curvature **within** ice .

P_0 can be calculated from the Clausius-Clapyron equation.*

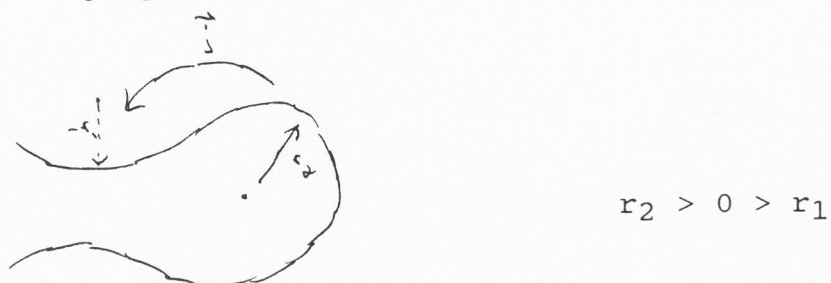
Using the above expression for P_1 and P_2 , we obtain the desired flux equation

$$j \approx -(D\alpha\delta^3P_0/T^2Rk\Delta x)(1/r_1 - 1/r_2) .$$

This flux equation explains the growth of large grains at the expense of small ones ($r_2 > r_1$), and the growth of 'necks' which sinter grains together ($r_2 > 0 > r_1$). (Fig. 2)



(i) growth of large grains at expense of small ones



(ii) sintering

Fig. 2 - Equitemperature processes

* for derivation of Clausius-Clapyron equation see:
"Handbook of Snow" (11) Ch. 7, or any good introductory
thermodynamics text.

It has been calculated that a particle of radius 1 μm would disappear in about one hour, while a particle of radius 10 μm would last about four days and a particle with radius 100 μm would last over one year.[♣] Thus, curvature effects appear to be important primarily when large differences in curvature are present.

Temperature-Gradient Metamorphism

Temperature gradients are commonly formed across the snowpack as a result of energy transfer at the ground and at the snow surface. Energy transfer at the ground is small but approximately constant. Typical values are 250-850 kJ/m^2 per day.[•] The flux will depend on soil type, water content, and whether or not the soil is frozen. During the winter, when the snowpack is not melting, the energy flux from the ground into the snow will maintain a temperature gradient across the deepest layers of the snow, and water vapor will diffuse from the soil into the bottom of the snowpack. For this reason, large TG grains are often found near the ground. These are often referred to as "depth hoar". Temperature gradients can also be found at other depths, depending on the rate of heat transfer at the surface. This flux is much more complicated than that at the ground and is discussed further in the next section (III;B).

[♣] "Dynamics of Snow and Ice Masses"; (1); p.311

[•] Ibid.; p.368

The process of TG metamorphism results from the diffusive transport of water vapor from warmer regions to colder regions within the snowcover. This diffusion occurs in a grain-to-grain manner (Fig. 3). The grains sublime at the top and grow at the bottom, resulting in pyramid-shaped grains. Since the thermal conductivity of ice is about 100 times that of air[■] and convective currents within the snow are essentially nonexistent, the temperature gradient across the air space between grains is much greater than that through the snowpack as a whole. (Fig. 3)

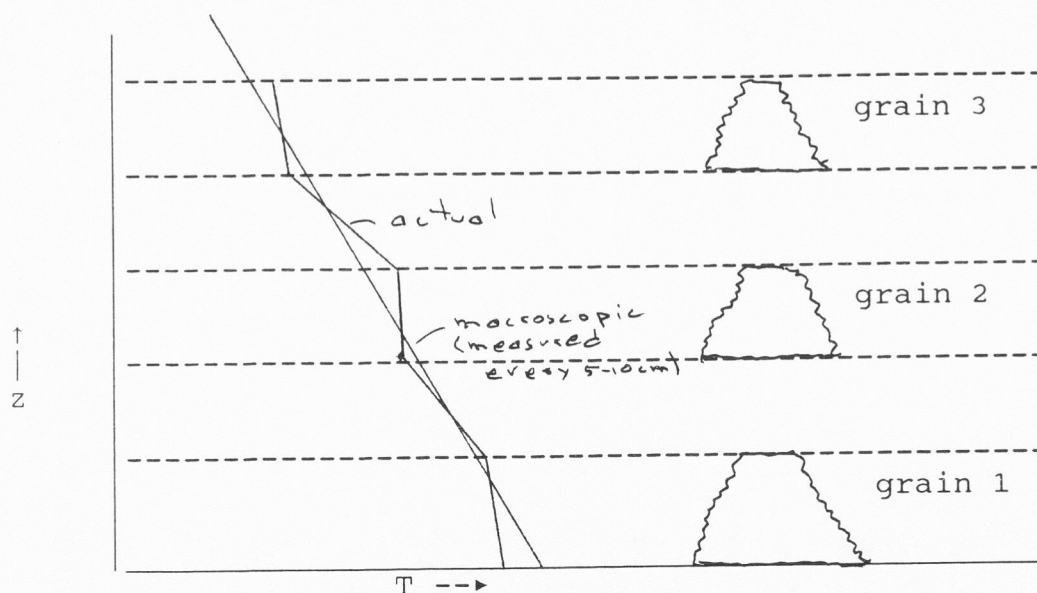


Fig. 3 - TG Metamorphism[○]

In a relatively uniform snowpack, it has been found that diffusion of water vapor is highest near the soil and steadily decreases towards the surface. Since the soil underneath is a major source of water vapor, mass transfer near the ground varies

■ $k(\text{air})=0.025 \text{ W/m K}$ $k(\text{ice})=2.2 \text{ W/m K}$
from "Handbook of Snow"; (11); p.101

○ adapted from Sommerfeld/LaChapelle; (22)

with the water content of the underlying surface. Vapor diffusion in upper layers appears to be approximately uniform.©

There is only limited correlation between vapor transfer and temperature gradient in the upper layers of the snowpack. However, the transfer of vapor from the soil into the snow depends greatly on the temperature gradient. At low gradients (0.07 to 0.09 ° per cm) the intensity of vapor flow is about 0.4 to 0.7×10^{-3} g/cm² per day over different soils. At a higher gradient (0.25 to 0.28 ° per cm) the transport intensity can vary from less than 1.5×10^{-3} g/cm² per day over well-drained areas to over 3.5×10^{-3} g/cm² per day over marshy areas.©©

In the grain-to-grain transport process, vapor is used in the formation of TG crystals at each step. Above about 30 cm most vapor from the soil has already been deposited on crystals and vapor diffusion is fairly uniform. In snow pits dug over an area with different underlying materials, such as rock and humus, a large difference in the amount and size of TG crystals just above the ground can often be observed. Such differences are usually limited to the lowest 30 cm or so.

The establishment and maintenance of a temperature gradient across the snowpack can be studied by considering the associated

© Kolomyts; "Diffusion Mass Transfer and Recrystallization"; in (2); p. 10 - 28

©© Ibid

heat transfer. In addition to conduction, heat transmission can also be attributed to the grain-to-grain vapor transport. When vapor evaporates from a lower grain and condenses on a grain above, the heat of sublimation is transferred also.

To derive the appropriate equation for heat transfer, we consider a small cylindrical volume between grains and equate the change in the amount of heat within this volume to the amount of heat entering (leaving) the volume. (Fig. 4).

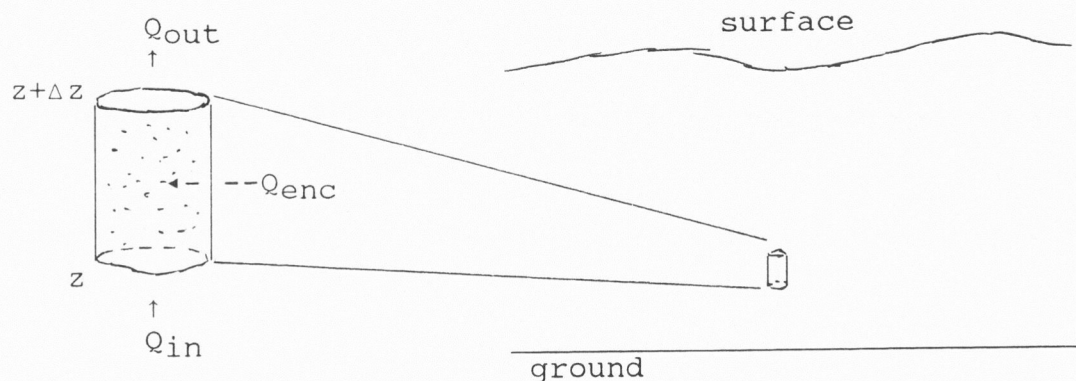


Fig. 4 - Heat Flow Between Grains

Assuming uniformity across the cylinder,

$$Q_{enc} = \int_z^{z+\Delta z} (c_p \sigma \Delta T + LAC) ds$$

and

$$Q_{in} = kA [T_z(z+\Delta z, t) - T_z(z, t)] \\ + DLA [C_z(z+\Delta z, t) - C_z(z, t)] ,$$

where:

σ = density

c_p = specific heat

T = temperature

k = thermal conductivity of dry medium

C = vapor concentration

D = diffusion coefficient of water vapor

D = diffusion coefficient of water vapor

L = latent heat of sublimation

z = distance from ground (normal)

A = cross section of cylindrical volume .

Setting $dQ_{enc}/dT = Q_{in}$, noting that c_p, σ, L, z , and $z+\Delta z$ are not functions of time within our volume, and cancelling the nonzero constant A results in

$$z^{z+\Delta z} (c_p \sigma \frac{dT}{dt} + L \frac{dC}{dt}) ds = k \left[\frac{dT}{dz} \right]_z^{z+\Delta z} + DL \left[\frac{dC}{dz} \right]_z^{z+\Delta z} .$$

Applying the mean value theorem on the left side, dividing the equation by Δz , and letting Δz shrink to zero gives the desired equation,

$$c_p \sigma \frac{dT}{dt} + L \frac{dC}{dt} = k \frac{d^2 T}{dz^2} + DL \frac{d^2 C}{dz^2} .$$

If vapor diffusion is negligible so that $dC/dt \approx 0$ and dC/dz is approximately constant, we are left with the Fourier equation

$$\frac{dT}{dt} = a \frac{d^2 T}{dz^2}$$

where: $a = k/\sigma c_p =$ thermal diffusivity .

The first equation has been solved numerically after using the ideal gas law and expressing pressure exponentially.⊙⊙

Comparison of the numerical results with the solution of the Fourier equation shows that the vapor transport mechanism is

important at low densities. As density increases, conduction becomes the dominant mechanism through which heat is transferred and the Fourier equation becomes a good approximation. This is due to the fact that the fraction of air present decreases. The Fourier equation is also accurate at low temperatures, since the amount of vapor becomes insignificant.

To describe the vapor pressure gradient, we can apply the Clausius-Clapyron equation,

$$dP/dT = L/T\Delta V$$

where: P = pressure

T = temperature

L = heat of sublimation

ΔV = difference in volume between solid and gas .

Neglecting the volume of the solid (i.e. assuming it is much less than that of the vapor) and treating the vapor as an ideal gas,

$$\Delta V \approx V_{\text{gas}} = RT/P$$

so

$$dP/dT = LP/RT^2$$

or

$$d(\ln P)/dT = L/RT^2 .$$

This can be integrated to find P as a function of T.☺♥

The pressure gradient with respect to distance z can be

☺♥ Denbigh; "Principles of Chemical Equilibria";
Cambridge, 1957; p 200

expressed:

$$\begin{aligned} d(\ln P)/dz &= d(\ln P)/dT \times dT/dz \\ &= (L/RT^2) (dT/dz) . \end{aligned}$$

The flux j and ultimately the rate of accumulation r of deposited ice can be found in terms of the temperature gradient.

Using Fick's Law and the ideal gas law we can write^{⊙♦}

$$j = -D/R\Delta z(P_1/T_1 - P_2/T_2) .$$

If $T_1 \approx T_2 = T$ then this can be written

$$\begin{aligned} j &= -(D/RT) (dP/dz) = -(D/RT) (P) (d(\ln P)/dz) \\ &= -(D/RT) (LP/RT^2) dT/dz . \end{aligned}$$

The rate of accumulation of ice will be equal to the rate of decrease of flux,

$$\begin{aligned} r &= - dj/dz \\ &= (DL/R^2T^3) [P d^2T/dz^2 + (dT/dz) (dP/dz)] . \end{aligned}$$

Using $dP/dz = (dP/dT) (dT/dz)$

and $dP/dT = LP/RT^2$

an expression for r is arrived at:^{⊙♣}

$$r = (DLP/R^2T^3) [d^2T/dz^2 + (L/RT^2) (dT/dz)^2] .$$

The second term is of particular importance for shallow snow in a

⊙♦ see preceding section on ET metamorphism

⊙♣ see "The Formation Rate of Depth Hoar"; (29) for a discussion on evaluating D and for relevant experimental results

in continental mountainous areas (such as the Rockies and the Wasatch) and are especially conducive to the growth of depth hoar.

The T^{-n} dependence in the preceding expressions is cancelled in part by D , which is proportional to $T^{1.7}$. The remaining negative exponent is more than offset by the exponential dependence of P on T . One can integrate the Clausius-Clapyron equation to find P as a function of T . An alternative is to use an empirical relation valid between 0°C and -20°C such as^{⊙♣}

$$P = P_0 (1 + T_C^{3/2}/21.5)^{-1}$$

where P_0 = saturation vapor pressure at 0°C

T_C = negative of mean snow temperature ,

or an approximation such as^{⊙•}

$$P(T) = 611 \exp [0.0857(T - 273)] \text{ N/m}^2.$$

The resulting expressions for J and r increase with T . This is in agreement with field observations. Vapor flux appears to drop by a factor of about 2 for every 8°C drop in temperature. Higher flux and growth rates are another reason why TG crystals, often large ones, are found just above the ground where the temperature is typically close to 273°K .

⊙♣ Ibid.

⊙• "Temperature-Gradient and Equi-Temperature Metamorphism of Dry Snow", (32)

B - Heat Exchange at the Surface

There are several means through which heat enters and exits the snowpack. Heat flows up from the ground into the snow, convection and latent heat exchange occurs at the surface, and radiation is emitted, absorbed, and reflected at the surface.

The ground heat flux is negligible when compared, on a daily basis, to the radiative and convective transfer which occur at the surface. It's primary effect is to provide a source of heat and water vapor which promote the growth of depth hoar at the base of the snowpack.☉ The effects of molecular transfer are smaller than those of radiative transfer, but are often important in determining the rate of melting. In the absence of wind, molecular heat transfer is very slow. With increased wind there is increased molecular contact due to mixing, and energy transfer rates increase by a factor of up to 10,000 times. Like the transport of snow by wind, the rate of heat transfer depends on the wind shear against the snow surface.

The radiation balance at the snow surface is usually the primary factor in heat exchange. Measurements of certain components of this balance can also indicate the surface conditions. The next two sections will focus on these radiative processes.

☉ see section on "Temperature Gradient Metamorphism"

Radiation Balance

The full-spectrum radiation flux is generally divided into two parts, a short-wave component and a long-wave component. Short-wave radiation consists of wavelengths from 0.2 to 3.0 μm , while the long-wave radiation falls into the 3.0 to 100 μm range. The short-wave radiation comes from the sun, while the long-wave component is emitted by the earth and the atmosphere.

The net long-wave radiation consists of an upward flux of energy emitted from the snow and a downward flux which comes from ozone (2%), carbon dioxide (17%) and water vapor (81%) in the atmosphere. The upward flux is generally greater, so that the long-wave balance results in a loss of energy from the snowpack. Both the upward and downward fluxes can be described by Stefan-Boltzman type laws. Snow emits as an almost perfect black body in the long-wave portion of the spectrum, so the loss of energy is

$$Q = \epsilon_S \sigma T_S^4$$

where ϵ_S = emissivity (0.97 - 1.0)

σ = Stefan-Boltzman constant

T_S = absolute temperature of snow surface .

The downward portion of the balance depends on the air temperature and vapor pressure 1.5 to 2.0 meters above the snow surface. The most common expression for this component was first

suggested by Brunt:⊙

$$Q = \sigma T_a^4 [a + b(p)^{1/2}] ,$$

where σ = Stefan-Boltzman constant

T_a = absolute air temperature 1.5 to 2.0 m above snow

p = vapor pressure 1.5 to 2.0 m above snow

a, b = empirical coefficients .

The coefficients a and b will depend on time of year and location. Measurements over the Russian plains and Canadian prairies indicate that a is between .6 and .7 and that b is about 0.005 .[⊙] Considerable scatter can be expected when applying this relation due to air temperature gradients and to vapor above 100m. For $p = 10\text{mb}$, $a = .68$, and $b = 0.005$ one obtains an effective emissivity of about 0.85.

In alpine areas, topographic features complicate calculation of the amount of radiation received at a point. A valley floor, for instance, receives less radiation from the sky but receives radiation from adjacent slopes. Another difficulty is obtaining accurate estimates of long-wave flux under cloudy or partly-cloudy skies as well as under vegetative cover. Partly because of these complications, the short-wave radiation balance is often of more interest. The difficulties are more tractable and measurements are easier to obtain.

⊙ Brunt, D.; "Physical and Dynamical Meteorology"; Cambridge Univ. Press 1952

⊙ "Handbook of Snow"; (11); Ch. 9

The short-wave component of radiation consists of sunlight, which falls within a very narrow wavelength band with maximum intensity at $0.47 \mu\text{m}$. The amount of solar radiation which reaches the earth's surface depends on latitude, season, time of day, topography, vegetation, cloud cover and atmospheric conditions. While passing through the atmosphere, radiation is reflected by clouds, scattered by air molecules, dust and particulates and absorbed by ozone, water vapor, carbon dioxide, and nitrogen compounds. The absorbed energy increases the temperature of the air, which increases long-wave emission.

A large portion of the short-wave radiation which reaches the snow surface is reflected. The 'albedo' is an indication of the amount reflected. The coefficient of reflection is also a function of snow crystal properties, allowing further conclusions to be drawn from the albedo, which is easily measured.

Albedo

The ratio of the amount of radiation reflected to the amount incident, in the short-wave spectrum, is called the albedo (A) of a surface. This is the simplest measurable parameter which yields information on the distribution of radiant energy in the snowpack. Since the albedo varies inversely with the square root of grain size,[⊙] it can provide information about metamorphism and melting in the top part of the snowpack.

[⊙] "Theory of the Optical Properties of Snow"; (30)

The albedo can be defined mathematically as:

$$A = \tau_1 f^{\tau_2} \int r(\tau) I(\tau) d\tau / \tau_1 f^{\tau_2} \int I(\tau) d\tau$$

where: $r(\tau)$ = reflectivity

$I(\tau)$ = incident monochromatic intensity

The limits τ_1 and τ_2 fall within the range of 0.2 to 3.0 μm and should reflect the characteristics of the measuring instrument.

At each ice-air interface within the snow an incident ray will be refracted and reflected. If the orientations of crystal faces are assumed to be random, the change in direction of the incident ray will be random. As a result, the direction of a light ray rapidly becomes uncorrelated with its initial direction. The path followed is analogous to Brownian motion, and is subject to Fick's second law of diffusion. Thus, assuming absorption does not occur before loss of directional correlation, we can apply Fick's second law of diffusion to the study of light within snow.

Assuming a homogeneous, isotropic medium,

$$dE/dt = D \nabla^2 E - kE$$

where E = density of radiant energy

D = diffusion coefficient

k = absorption rate .

For a one dimensional, steady state problem this becomes

$$d^2E/dz^2 = b^2E ; b = (k/D)^{1/2}$$

and $E = C_1 e^{-bz} + C_2 e^{bz}$.

Since energy will decrease with penetration, the solution will be: $E = E_0 e^{-bz}$ where $E_0 = E(\text{surface})$.

The albedo is the ratio of the upward radiation at the surface to the total flux (up and down) at the surface. If we assume that the upward radiation is proportional to the total density of radiant energy (E_0) by a factor of τ and use Fick's first law for the downward flux we obtain

$$A = \tau E_0 / [-D(dE/dx)_{x=0} + \tau E_0] .$$

For the exponential attenuation obtained above

$$A = 1/1 + \Omega \quad \text{where } \Omega = (kD)^{1/2}/\tau \quad (\text{dimensionless}) .$$

This model of exponential decay should be valid for albedos of down to 0.8. It is also easily extended to a layered snowpack by taking the full solution to Fick's second law for each layer and using continuity at the layer interfaces for boundary conditions.⊕⊕

The extinction coefficient b depends on wavelength, particle size, snow density, and foreign matter content. The relative importance of these factors is not known and a satisfactory functional relation for b has not yet been found. Absorption is high in the red/infrared portion of the spectrum, while albedo is highest in the UV portion. Albedo decreases as the snow surface ages. In deep mountainous snowpacks, metamorphic processes produce gradual structural changes which affect scattering characteristics. For shallow snowpacks (less than 25 cm) albedo

⊕⊕ see "Diffusion Theory Applied to Radiant Energy Distribution and Albedo of Snow"; (31)

will decrease rapidly and, after about one week, will level off near 20%. This is due to the influence of the lower albedo of the underlying surface, since typical maximum penetration depths are 20 - 30 cm.☉♥ Values of albedo typical of a variety of snow conditions are given in Fig. 4.

Compact, Dry, Clean 85%
 Clean, Wet, Fine 65%
 Clean, Wet, Granular 62%
 Porous, Wet, Greyish 50%
 Porous, Dirty, Saturated 35%

Fig. 5 - Albedo; Solar Angle = 32⁰

Surface Hoar

The growth of surface hoar is an interesting and important type of surface energy exchange. During the day the air above the snow may contain an appreciable amount of water vapor. This air may become saturated with respect to the snow surface at colder evening temperatures. The vapor will then condense on the surface in the form of surface hoar, the solid equivalent of dew. Layers of surface hoar are particularly weak and cohesionless. These properties can persist for a long time after the layer is buried.

The conditions under which surface hoar forms occur mainly during cold clear nights. The crystals are sparkling and angular,

☉♥ Samukashvili in 'Physics of Snow, Avalanches, and Glaciers'; (2)

similar in some respects to depth hoar. Growth conditions usually result in a plate-like structure a few millimeters thick but 50mm or more in diameter. However, other forms can occur under appropriate conditions.❖

Due to radiational cooling, a very large temperature gradient can form at the snow surface (2°C or greater). Despite this large gradient, a steady-state analysis based on conservation of mass and momentum predicts that diffusive flux alone cannot account for the amount of condensate.❧ This suggests that convection plays a major role. Consistent with this is the observation that growth of surface hoar will not occur in the presence of any detectable horizontal air motion.

Snow albedo exhibits a diurnal hysteresis (about 4% of average) which may be attributed, to some extent, to surface hoar. The albedo in morning has been found to be higher than that in the afternoon for identical solar elevation. It has been suggested that this is primarily due to a thin layer of surface hoar formed during the night which is removed as the temperature increases during the day.❧ Surface irregularities, such as sastrugi, may contribute to some degree, but would presumably appear with a period of 12 hours rather than 24.

❖ see "Crystal Orientation" section; Part I

❧ "Studies on Surface Hoar"; (45)

❧ "Diurnal Hysteresis of Snow Albedo"; (17)

IV - Mechanics

The mechanics of a seasonal snowpack are generally not conducive to rigorous analysis. Snow cover is neither homogeneous nor isotropic, and properties vary widely both spatially and temporally. When additional snow falls, stresses change and boundaries move. Initial properties of deposited snow depend on the size, shape and temperature of the crystals as well as the packing arrangement. In calm conditions, fresh snow can be deposited with densities as low as 0.05 g/cm^3 . On the other hand, windslab densities can exceed 0.3 g/cm^3 .

Once deposited, metamorphism changes the mechanical properties as the crystals are altered. These changes can strengthen or weaken various strata within the snowpack. At the same time, the snow is under the influence of body forces. Volumetric strains are usually compressive, resulting in an increase in density with time. If the snow is on an incline, it is also continually creeping downhill.

Despite these difficulties, some attempt needs to be made to arrive at an acceptable analysis if our understanding of avalanches is to progress. We need to develop a model for the properties of snow which is manageable but does not sacrifice too much complexity to be realistic. Then general equations for stress, strain and failure can be developed. These general equations can be simplified for certain special cases of

interest. For more complicated applications, complex variable or numerical methods can be applied.

A - Viscoelastic Properties

Snow is a viscoelastic material in which the viscosity is nonlinear with stress. In an elastic material stress is a function of strain. An elastic material 'remembers' its initial state and will return to that state when external forces are removed. In a viscous material stress is a function of strain rate but not of strain. Thus it will not 'remember' any initial or reference state. Viscoelastic materials exhibit the effects of both elasticity and viscosity. Stress is a function of the past history of strain. These materials have an imperfect 'memory' of limited duration. They are sometimes called rate-sensitive materials.

For elastic behavior

$$\sigma = E\epsilon \quad \text{or} \quad d\epsilon/dt = (1/E)d\sigma/dt .$$

For viscous fluids which are linear

$$\sigma = \lambda d\epsilon/dt \quad \text{or} \quad d\epsilon/dt = \sigma/\lambda$$

where λ is the 'viscous traction'.

Summing the strain rates for elastic and viscous behavior results in a viscoelastic rheological model known as the Maxwell body. The resulting equation is

$$d\epsilon/dt = (1/E)[d\sigma/dt + \sigma/\Gamma] ; \quad \Gamma = \lambda/E .$$

Integrating this for constant stress, $d\sigma/dt = 0$, gives

$$\epsilon = \sigma t/E\Gamma + \epsilon_0$$

$$= (\sigma/E)(1 + t/\Gamma) = (\sigma/E) C(t)$$

where $\epsilon_0 = \sigma/E$ is the instantaneous strain occurring when the constant stress is first applied. This is the simplest "creep equation" and $C(t)$ is called the "creep function".

If we consider an initially unstrained Maxwell body which is strained by amount ϵ_0 for $t \geq 0$, the stress is found from

$$d\sigma/dt + \sigma/\Gamma = 0 ,$$

the solution of which is

$$\sigma = \sigma_0 e^{-t/\Gamma} = \sigma_0 \phi(t) .$$

The function $\phi(t)$ is known as the relaxation function, and Γ serves as a characterization of the materials 'memory'.

The stress-strain relation for a Maxwell body undergoing strain at a constant rate, $d\epsilon/dt = c$, can be obtained by integrating the defining viscoelastic expression above:

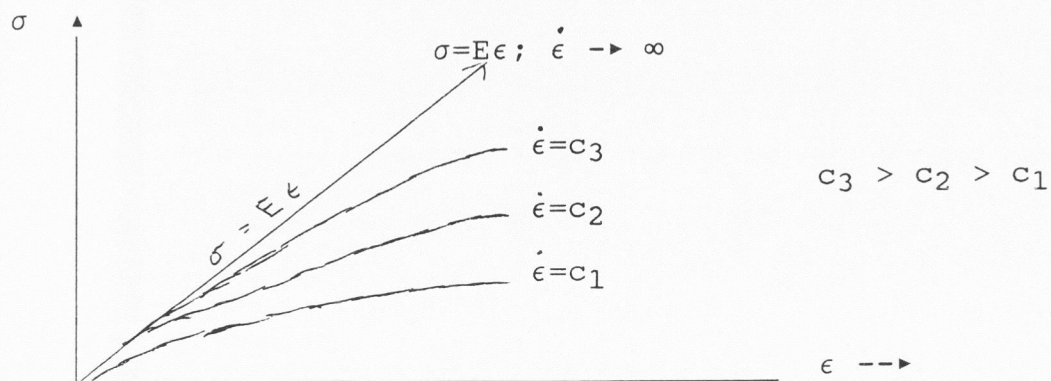
$$\sigma = c [1 - e^{-\epsilon/c\Gamma}] .$$

From this, we see that a stress-strain diagram will be rate-dependent.

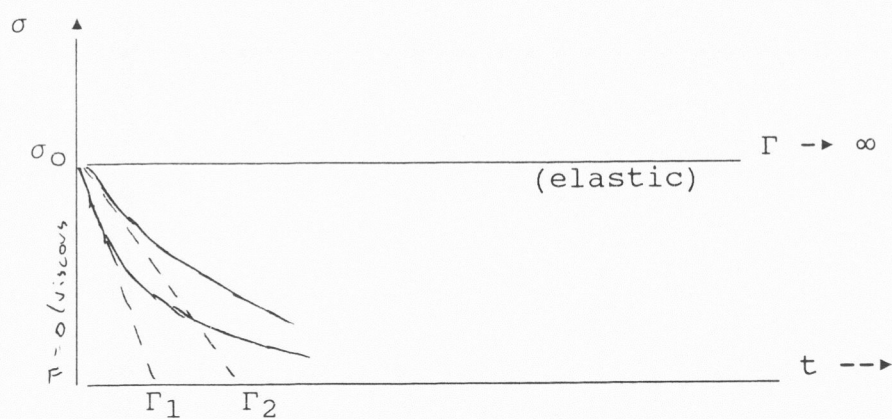
To summarize Maxwell body behavior:

- 1) Creep occurs at constant stress
- 2) Stress relaxation occurs at constant strain
- 3) The ϵ - σ diagram will vary with strain rate

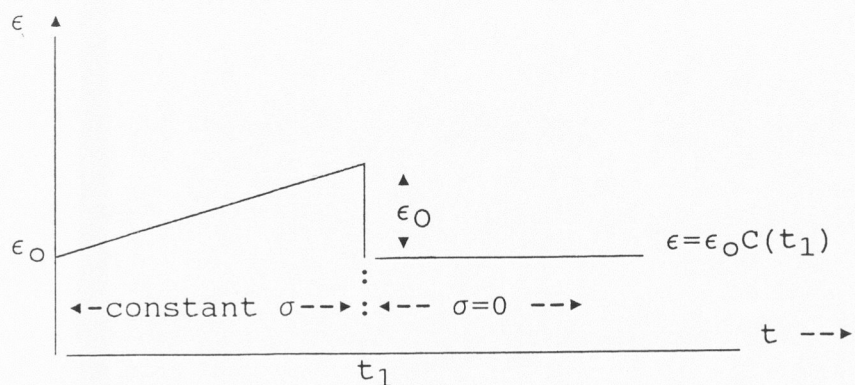
These properties are shown graphically in Fig. 1.



iii) Stress-strain behavior for constant strain rate



ii) Relaxation under constant strain



i) Creep under tension

Fig. 1 - Behavior of a Maxwell body

Summing the stresses for elastic and viscous behavior results in the rheological model for the Kelvin-Voight body. The viscoelastic equation for this model is

$$\sigma = E[\epsilon + \Gamma d\epsilon/dt] .$$

This exhibits no stress relaxation, since it reduces to the elastic expression for constant strain. However, for a constant stress σ_0 , the solution for ϵ is

$$\epsilon = (\sigma_0/E)[1 - e^{-t/\Gamma}] = (\sigma_0/E) C(t)$$

Here the creep function indicates that elastic deformation will be retarded (delayed). This is illustrated in Fig. 2.

After the removal of a constant stress the deformation decreases asymptotically and is recovered completely in the limit:

$$\epsilon = \epsilon(t_0)\exp[-(t-t_0)/\Gamma] ,$$

where t_0 is the time at which the constant stress is removed.

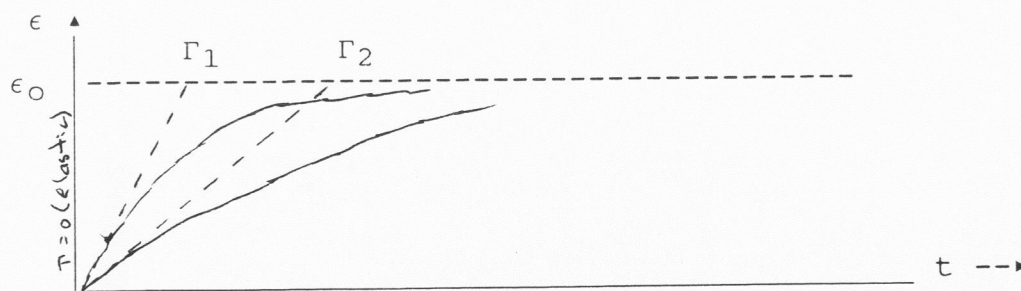


Fig. 2 - Kelvin-Voight Creep

Snow is represented reasonably well by a model which combines the Maxwell and Kelvin-Voight rheological models. When these two are combined in series, the resulting model is called the Burger's body. (Fig. 3) This is the model most commonly used to represent snow.

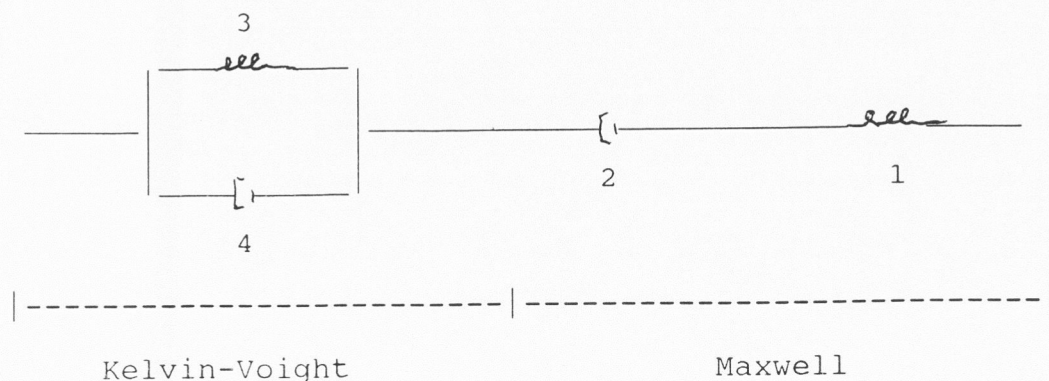


Fig. 3 - Burger's Body

When a fixed load is applied to snow, there is an instantaneous elastic deformation (1). This is followed by decelerating primary creep, or delayed elasticity, which is largely recoverable by relaxation (3,4). Finally an unrecoverable secondary creep occurs (2).

The main shortcoming of the Burger's body model is that it fails to account for the nonlinearity of secondary creep when stress is allowed to vary between wide limits. At low stress, the secondary creep appears to be Newtonian in nature. As stress increases, strain rate becomes dependent on higher powers of stress.

The secondary creep appears in the Maxwell portion of the Burger's body model. A modification including dependence on higher powers of stress is not uncommon in the Maxwell model. Other materials also exhibit this dependence, such as metals at

high temperatures.

The viscous strain rate can be replaced by

$$d\epsilon/dt = (\sigma/\sigma^*)^n \sigma/\lambda \quad \text{where } \sigma^* = \text{constant} .$$

Then the viscoelastic equation becomes

$$d\epsilon/dt = (1/E) [d\sigma/dt + \sigma/\Gamma(\sigma/\sigma^*)^n] .$$

The creep function for constant stress is now

$$C(t) = (\sigma/E) [1 + (t/\Gamma)(\sigma/\sigma^*)^n] ,$$

which results in a higher creep strain.

It often turns out that the behavior of snow for particular problems of interest is predominantly either elastic or viscous. In these cases a great deal of simplification is possible. When disturbances of short duration are considered, behavior is generally elastic. Where sustained loads are of interest, viscous strains will usually be much greater.

B - Plane Strain Equations

Most avalanche slopes are essentially 'flat', with radii of curvature typically greater than 100m.⊙ General stress-strain relations will reduce to a manageable number of equations under the assumption of plane-strain. This simplification should be valid as long as surface gradients and snow thickness variations in a direction perpendicular to the fall line are negligible.

Since the analysis here is intended to apply to viscous deformation under sustained loading, we will treat the snow as a viscous material. The coordinate system shown in Fig. 4 will serve as the basis for the equations through the remainder of this section.

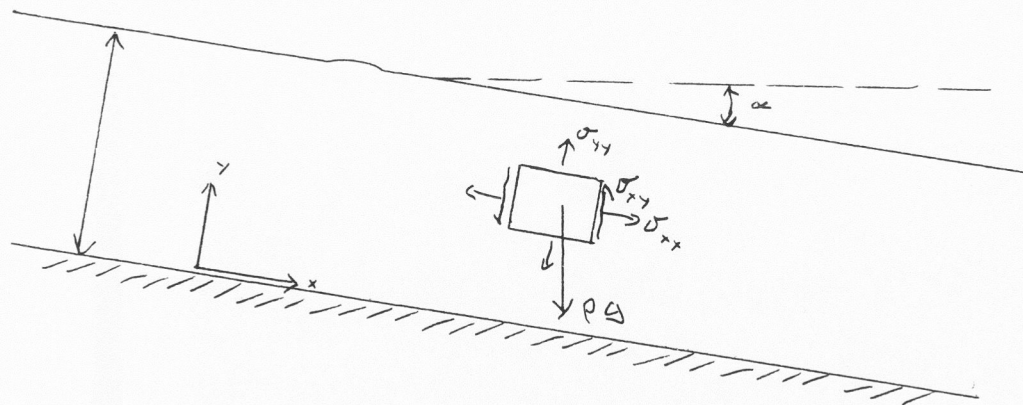


Fig. 4 - Coordinates

⊙ based on published observations and on photographs; for references see "Dynamics of Snow and Ice Masses"; (1); Ch. 7

Analysis of all plane strain problems in cartesian coordinates is based on the following equations. They are derived by assuming that there are no stress gradients, strains, or velocities in the z direction (perpendicular to fall line along slope).[⊙] In these equations μ is shear viscosity and η is bulk viscosity.

i) Quasi-equilibrium (acceleration ≈ 0):

$$d\sigma_{xx}/dx + d\sigma_{yy}/dy + \rho g_x = 0 \quad (g_x = g \sin \alpha)$$

$$d\sigma_{xy}/dx + d\sigma_{yy}/dy + \rho g_y = 0 \quad (g_y = g \cos \alpha)$$

ii) Stress-strain and strain-rate/velocity:

$$\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 3\eta[\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy}] = 3\eta[du_x/dx + du_y/dy]$$

$$2\sigma_{xx} - \sigma_{yy} - \sigma_{zz} = 2\mu[2\dot{\epsilon}_{xx} - \dot{\epsilon}_{yy}] = 2\mu[2du_x/dx - du_y/dy]$$

$$2\sigma_{yy} - \sigma_{xx} - \sigma_{zz} = 2\mu[2\dot{\epsilon}_{yy} - \dot{\epsilon}_{xx}] = 2\mu[2du_y/dy - du_x/dx]$$

$$2\sigma_{zz} - \sigma_{xx} - \sigma_{yy} = -2\mu[\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy}] = -2\mu[du_x/dx + du_y/dy]$$

$$\sigma_{xy} = 2\mu\dot{\epsilon}_{xy} = \mu[du_x/dy + du_y/dx]$$

iii) Compatibility:

$$d^2\dot{\epsilon}_{xx}/dy^2 + d^2\dot{\epsilon}_{yy}/dx^2 = 2d^2\dot{\epsilon}_{xy}/dxdy$$

Other useful forms of these expressions are:

iv) Strain-rate:

$$\dot{\epsilon}_{xx} = du_x/dx = (1/9\eta\mu)[(3\eta+\mu)\sigma_{xx} - \frac{1}{2}(3\eta-2\mu)(\sigma_{yy} + \sigma_{zz})]$$

$$\dot{\epsilon}_{yy} = du_y/dy = (1/9\eta\mu)[(3\eta+\mu)\sigma_{yy} - \frac{1}{2}(3\eta-2\mu)(\sigma_{xx} + \sigma_{zz})]$$

⊙ The equations are written here in a form which closely parallels that found in "Avalanches"; (7). For the development of the plane strain relations see this reference or an introductory text on Mechanics of Solids, such as (46).

v) Normal stress:

$$\sigma_{xx} = (1/3) [(3\eta+4\mu)\dot{\epsilon}_{xx} + (3\eta-2\mu)\dot{\epsilon}_{yy}]$$

$$\sigma_{yy} = (1/3) [(3\eta-2\mu)\dot{\epsilon}_{xx} + (3\eta+4\mu)\dot{\epsilon}_{yy}]$$

$$\sigma_{zz} = (1/3) [3\eta-2\mu] (\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy})$$

$$\sigma_{zz} = [(3\eta-2\mu)/2(3\eta+\mu)] (\sigma_{xx} + \sigma_{yy})$$

C - Analysis of an Inclined Snowpack

The easiest place to begin is with a long uniform slope, a situation for which boundary conditions are simple. For cases which are not so straight-forward, a framework for numerical solutions can be developed.

Long Uniform Slope

This model is for a section of snow slope where there is no curvature along the fall line. It is sometimes called a 'neutral zone' problem since it serves as a model for the central region of the slab, away from the boundaries.

The quasi-equilibrium equations can be integrated directly after making the assumptions $d\sigma_{xx}/dx=0$ and $d\sigma_{xy}/dx=0$,

$$\sigma_{xy} = -\rho g y \sin \alpha$$

$$\sigma_{yy} = -\rho g y \cos \alpha .$$

The normal stress relations for σ_{xx} and σ_{zz} with $\epsilon_{xx}=0$ become

$$\sigma_{xx} = \sigma_{zz} = (1/3)[3\eta - 2\mu]\dot{\epsilon}_{yy} .$$

Using the normal stress relation and the quasi-equilibrium equations for σ_{yy} gives us

$$\sigma_{xx} = \sigma_{zz} = -[(3\eta - 2\mu)/(3\eta + 4\mu)] \rho g y \cos \alpha$$

The strain rates are

$$\dot{\epsilon}_{xx} = \dot{\epsilon}_{zz} = 0$$

$$\dot{\epsilon}_{yy} = -[3/(3\eta+4\mu)] \rho g y \cos \alpha = du_y/dy .$$

From the strain-rate/velocity equation for $\dot{\epsilon}_{xy}$ with $du_y/dx = 0$,

$$\dot{\epsilon}_{xy} = (1/2\mu)\sigma_{xy} = -(1/2\mu) \rho g y \sin \alpha = \frac{1}{2} du_y/dx .$$

The components of creep velocity are found from integration of the last two equations with the boundary conditions $u_x=u_y=0$ at $y=0$,

$$u_x = (1/2\mu)[g \sin \alpha] y^2$$

$$u_y = -[3/2(3\eta+4\mu)](g \cos \alpha) y^2 .$$

General Methods

The plane-strain equations can be reduced to a problem involving one stress function. This equation can then be solved numerically or, if boundary conditions allow, complex variable methods. We begin by defining a potential function V in terms of the specific body force,

$$-\nabla V = f .$$

Using the basic plane-strain equations, we obtain[♥]

$$[(3\eta+4\mu)/(6\eta+2\mu)] \nabla^2(\sigma_{xx} + \sigma_{yy}) = \nabla^2 V .$$

Now it is desirable to introduce a stress function Φ such that the quasi-equilibrium conditions are satisfied. The appropriate function is known as the Airy stress function and is defined

[♥] for an outline of the steps, see 'Avalanches'; (7)

$$d^2\phi/dx^2 = \sigma_{yy} - V$$

$$d^2\phi/dy^2 = \sigma_{xx} - V$$

$$-d^2\phi/dxdy = \sigma_{xy} .$$

Introducing this into the preceding equation,

$$\nabla^4\phi = [-6\mu/(3\eta+4\mu)] \nabla^2V .$$

If the body forces are limited to gravitational force ($f = g$) then $\nabla^2V=0$ and the Airy stress function will be a solution to the biharmonic equation

$$\nabla^4\phi = 0 .$$

Stresses in the snowpack can also be analyzed by using finite element methods. Some results of such analyses are discussed in the following section on slab failure.

D - Failure

When snow fails, the rupture process may be either ductile or brittle. Although the end result is similar, the development of stresses and strains leading to failure may differ substantially. For typical avalanche conditions, it is estimated that brittle failure requires loading rates exceeding $1 \text{ kg/cm}^2\text{-s}$ or strain rates exceeding $10^{-2} /\text{s}$.[♦] These estimates indicate that failure under body forces is initially ductile. However, this initial failure may be followed by brittle failure of the remaining anchors.

Various processes leading to slab release have been proposed.[♣] Avalanches released naturally probably experience ductile shear failure along a weak layer in the snow (the bed) followed by brittle failure along the crown and flanks. Avalanches which are triggered (by a skier or cornice fall, for example) may first fail along the crown in a brittle manner.

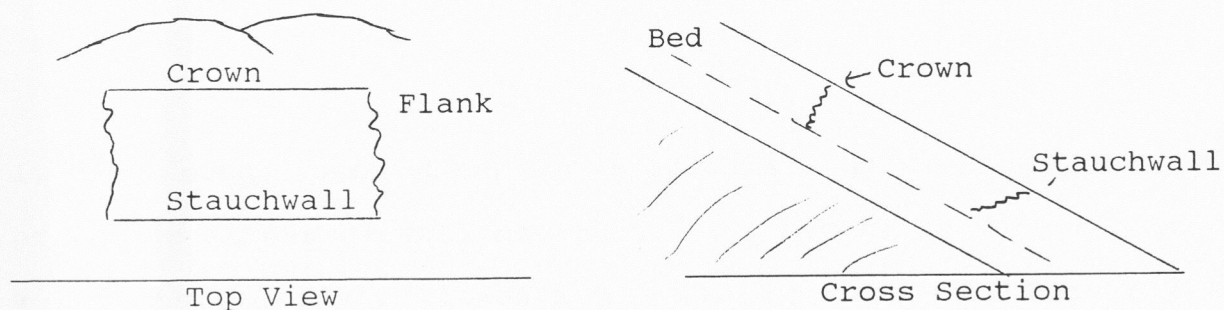


Fig. 5 - Nomenclature for Slab Fracture Surfaces

♦ Ibid.

♣ For a brief summary with references see introduction to: "The Role of Stress Concentrations in Slab Avalanche Release"; (50)

Studies of failure under both tension and shear have provided interesting and useful insight into the mechanical behavior of snow.

Shear Failure

It is suspected, based on theoretical calculations, that initial failure is usually in shear along the bed surface.[♣] This is followed by tensile failure at the crown. However, conditions leading to slab release vary so widely that it is highly doubtful that this is true in every case. The importance of tensile failure is discussed in the next section.

Stresses in the snow have been studied with finite element analysis using a triangular constant strain mesh.[•] The snowpack is treated as consisting of several isotropic layers, the properties of which are representative of a particular avalanche path or paths. The results can then be compared to events in these paths.

It has been found that as Poisson's ratio increases, the principle stress directions rotate in a clockwise manner. At $\nu = 0.45$, the principle elements all lie at an angle of nearly 45° with respect to the slope of the layer below the observed fracture line. Since the plane of maximum shear is 45° from the

[♣] see "Dynamics of Snow and Ice Masses"; (1); Ch. 7

[•] see (6), (48), (49)

maximum principle stress, shear failure along the bottom of a layer appears to be a likely event.■

A shear strength envelope (rupture stress vs. density) is available for snow.○ The shear stresses in low density layers falls within this envelope, indicating that shear failure may be initiated in lower density layers. However, the correlation between density and strength should not be relied upon too heavily, since these finite element models do not account for the type of snow crystals forming a layer. In equitemperature layers strength correlates fairly well with density, but in temperature gradient layers the correlation is poor.

Tensile Failure

While theories in which shear failure occurs first have gained prominence in recent years, tensile fracture continues to be a topic of great importance. This type of failure will always occur along the crown, either following shear failure at the bed or as the initial event in instances such as skier initiated avalanches.

Finite element analysis has predicted that stresses tangential to the slope (σ_{xx}) will be tensile only in the region about the

■ see "Material Property and Boundary Condition Effects on Stresses in Avalanche Snowpacks"; (6)

○ see "Snow Mechanics"; (5)

crown and only in the upper layers. They are compressive elsewhere. In some cases the maximum tension is actually above (upslope from) the crown fracture.⊙ This casts doubt on theories which predict that tensile failure at the crown precedes shear failure. Such a sequence appears to be likely only if something external initiates a fracture in the region of tension.

One mechanism leading to slab release is the collapse of a layer of depth hoar near the bottom of the snowpack. When conditions are conducive to this, skiers will often notice widespread collapse under their weight, even on flat terrain. This condition has been simulated by setting Young's modulus for the lowest layer to about one tenth its value in the other layers, allowing large deformations in the bottom layer. It was found that the shear stresses were similar in variation and magnitude to the 'baseline' runs, but that the tangential stresses increased greatly in magnitude in the crown region.⊙⊙ Thus avalanche release due to collapse is probably initiated by tensile failure at the crown.

Under high stress rates, snow behaves as an ideal brittle material. The tension failure at the crown exhibits little plastic deformation during rupture (i.e. the pieces could be fit back together). This implies that some of the extensive work in

⊙ see "Elastic Stresses in Layered Snow Packs"; (48)

⊙⊙ Ibid.

brittle fracture may apply to snow.⊙⊙

Griffith's theory for elastic cracks states that a crack will propagate when the elastic energy released by the infinitesimal extension of a sharp crack is equal to or greater than the specific surface energy of the newly formed surfaces. The minimum fracture stress is given by

$$\sigma_0 = [2\tau E/\pi c]^{\frac{1}{2}}$$

where τ = specific surface energy

E = Youngs modulus

c = crack length .

Since $\sigma_0 \propto c^{-\frac{1}{2}}$ the crack will become self-propagating once it reaches a critical length (stress remaining constant).

Many isotropic brittle solids exhibit similar fracture markings, and these markings can be identified on the surfaces remaining after a slab release. The initial fracture surface is smooth and is called the "mirror zone". This is followed by the "mist zone" which has fine features, and finally the "hackle zone" with its coarser features. The coarser features arise when the fracture has reached a limiting velocity (0.6 times the transverse wave velocity). Additional elastic energy no longer accelerates the fracture, but causes local branching of the fracture front.

⊙⊙ for further references on brittle fracture, see "The Role of Stress Concentration in Slab Avalanche Release"; (50)

V - Avalanches

A snow avalanche occurs when stress exceeds strength somewhere in the snowpack and a mass of snow moves down a slope. During snowstorms heavy precipitation and severe wind loading can cause stress within the snow to build up faster than strength is gained through compaction and sintering. Many avalanches occur during or shortly after a major storm. Other avalanches are released by internal triggers. Metamorphism, free water lubrication of buried crusts, and other changes within the snowpack can cause a decrease in strength. Externally triggered avalanches occur when a marginally stable mass of snow is jarred into motion by the application of an external force such as a falling cornice or a skier.

Three things are often considered to be necessary for an avalanche to occur. They are the existence of a strong cohesive layer (the slab), a weak layer, and a bed surface to slide on. The last does not appear to be necessary in all cases. If the snow releases within a thick layer of depth hoar, it can form a hard bed surface as it moves.

The avalanche path is generally divided into three regions. The 'starting zone' is where the failure process begins (i.e. the slab). The 'track' is the path the snow takes down the incline. In the track, any mass left behind is more than compensated for by the entrainment of additional mass. The 'runout zone' is where everything comes to rest.

A - Classification

Schemes for classifying avalanches serve several purposes. They allow meaningful information to be exchanged concerning incidents and control measures. They also allow events to be grouped in a way which is conducive to statistical analysis.

A widely accepted scheme is that proposed in 1973 by the Working Group on Avalanche Classification of the International Commission on Snow and Ice. This 'morphological' classification system is based on the characteristics of the three zones (starting, track, and runout). It is shown in Fig. 1.

Zone	Criterion	Alternative characteristics and symbol designation	
Origin (Starting)	A. Manner of starting	A1 starting from a point (loose snow avalanche)	A2 starting from a line (slab avalanche) A3 soft A4 hard
	B. Position of sliding surface	B1 within snow cover (surface layer avalanche)	B4 on the ground (full-depth avalanche)
		B2 (new snow fracture) B3 (old snow fracture)	
C. Liquid water in snow	C1 absent (dry snow avalanche)	C2 present (wet snow avalanche)	
Transition (Track)	D. Form of path	D1 path on open slope (unconfined avalanche)	D2 path in gully or channel (channelled avalanche)
	E. Form of movement	E1 snow dust cloud (powder avalanche)	E2 flowing along the ground (flow avalanche)
Deposit (Runout)	F. Surface roughness of deposit	F1 coarse (coarse deposit) F2 angular blocks F3 rounded clods	F4 fine (fine deposit)
	G. Liquid water in snow debris at time of deposition	G1 absent (dry avalanche deposit)	G2 present (wet avalanche deposit)
	H. Contamination of deposit	H1 no apparent contamination (clean avalanche)	H2 contamination present H3 rock debris, soil H4 branches, trees

Fig. 1 - Morphological Classification

The United States reporting system is used to record incidents in this country. It is described in Fig. 2.

<i>Symbol</i>	<i>Type avalanche</i>		<i>Size (based on volume of snow for the path in question)</i>	
HS	Hard slab	1	Sluff	(Any slide running less than 150 feet (50 m) slope distance regardless of other dimensions)
SS	Soft slab			
WS	Wet slab			
L	Loose	2	Small	(Relative to the path)
WL	Wet loose	3	Medium	(Relative to the path)
		4	Large	(Relative to the path)
		5	Major or maximum	(Relative to the path)
	<i>Trigger (activating agent)</i>			
N	Natural			
AS	Artificial, ski			
AE	Artificial, hand charge	G		<i>Running surface</i> Avalanche ran to ground in the starting zone
AA	Artificial, artillery	O		Avalanche ran on an old snow surface in the starting zone
AL	Artificial, avalauncher			
AO	Artificial, other			
				<i>Airblast</i>
HS-AA-2-G	Hard slab avalanche released by artillery. It was of small size and ran to ground in the starting zone.	J		Airblast was observed with the avalanche.
SS-AE-4-O-J	Soft slab avalanche released by hand charges. It was large for that path and ran on an old snow layer. Airblast was observed.			

Fig. 2 - U.S. Reporting System

In western Canada, avalanches are also given a number corresponding to a nominal size classification scheme. This is described in Fig. 3.

- size (1) - sluff; any small amount of snow which would not injure a human,
- size (2) - could injure a human,
- size (3) - could damage buildings, automobiles, break a few trees;
- size (4) - could destroy large vehicles, or forests with areas up to 4 ha, and
- size (5) - unusual, catastrophic events, which could damage villages or destroy large areas of forest.

Fig. 3 - Classification by Magnitude

B - Flow Dynamics

Dynamics involves the study of such parameters as velocity, mass, runout distance, and impact forces. An understanding of these parameters is necessary to facilitate the effective design of defense structures and adequate preparation of zoning plans.

After a slab releases, the dislodged snow will accelerate rapidly. As it moves downhill it will accumulate mass by breaking up the snow in its path. Initial motion will be of a rolling/gliding type. As velocity increases the motion will become turbulent. While the slab initially moves as one mass, it is broken into large blocks quickly. These blocks break up into masses of rounded and pulverized material. In dry snow, fine particles mix with air to form a powder.

Snow moving along the ground is called a 'flowing' avalanche. When snow is carried by turbulent air motion it is called a 'powder' avalanche. Both forms are often present. Most of the momentum is probably concentrated in a flowing layer 1-10 m thick. While a dust cloud can extend upwards to about 100m, it is typically of low density. These comments on momentum are supported by observations of damages to structures and vegetation.

When the slope becomes less steep, the moving snow mass decelerates, eventually coming to rest. If the velocity is high enough when this slowing process begins, the runout zone may be very long. In extreme cases, it may extend uphill on a slope facing the avalanche path.

Observations

Field observations of moving avalanches can provide qualitative information on the nature of flows as well as hard data on the velocities and forces involved. Visual observation of naturally occurring avalanches is difficult, since it involves being in the right place at the right time. However, slides are often intentionally released using explosives to control paths which threaten highways or ski areas. These control releases are easily observed. Instruments to measure velocity and force can be put in place in any path during a period of low risk and recovered later.

Stereographic surveys carried out in the Khibin ranges in the Soviet Union reveal a characteristic pulsation of the front velocity in both wet and dry snow avalanches. The photographic images indicate that these pulses are not generally due to topographic changes of slope. Thus these changes in velocity which recur with a certain quasi-periodicity are probably indicative of the physical processes occurring in the moving snow. Velocity pulsation of the front may be observed for the

duration of motion, the period averaging 4 - 6 sec.⊙

Avalanche velocities have been measured using photogrammetry, mechanical switches, and hand timing. Maximum measured speeds are 50-55 m/s.⊕ Calculations based on indirect evidence and on theoretical models indicate that speeds in excess of 100 m/s may be obtained in large paths. Mass elements within a moving snow mass may be moving much faster than the overall avalanche velocity. Since the flow is turbulent, velocities within should be statistically distributed and high speed transients should exist. Transient velocities up to 150 m/s have been recorded using pressure sensors.♥

Investigators have measured impact forces over three winters in the Kurobe gorge of the Japanese Alps. The first winter their equipment was swept away, and they estimated that the forces exceeded 47 Mg/m². The second winter was a light one by area standards, and forces between 101 and 139 Mg/m² were observed. The third winter was a heavy one, resulting in a maximum impact force of 201 Mg/m² and a maximum impulse of 389 Mg s/m². The maximum excursion of atmospheric pressure found was -21 mbar.♦

⊙ see Bryukhanov in "Physics of Snow, Avalanches, and Glaciers"; (2)

⊕ "Dynamics of Snow and Ice Masses"; (1); Ch. 7, Table 4

♥ Shimizu, et. al.; Low Temp. Sci., Ser. A, Vol. 35; (1977)

♦ "Snow Avalanches: A Review"; (40). The values for force and pressure quoted are from this article.

Observations of avalanches, both direct and indirect (through damages and effects), indicate that avalanches are often preceded by an air blast. This is sometimes mistakenly referred to as a shock wave. Since even the highest avalanche velocities are subsonic, it is doubtful that this air blast is truly a shock wave. However, since the sonic velocity in a snow-air mixture of certain densities may be less than observed avalanche velocities, it appears to be possible for a shock wave to propagate within a flowing avalanche.♣

Theory

The problem of computing avalanche velocities has been approached in many different ways over the past forty years. However, they can be grouped into two categories. The snow may be treated as a unified mass in motion, or it may be considered to be a flowing fluid.

The equation of motion for a unified mass follows directly from Newton's laws and is

$$m(dV/dt) = mg (\sin \beta - \mu \cos \beta),$$

where β is the angle of the slope and μ is the coefficient of friction. For motion which is not gliding in nature, another term can be added to account for air drag and plowing forces.♠

♣ Bryuckhanov, A.V., et. al.; "On Some New Approaches to the Dynamics of Snow Avalanches"; (1967); in '1966 Proceedings, Vol. 1, Inst. of Low Temp. Sci., Hokkaido Univ.'

♠ see Ch. 7 of 'Dynamics of Snow and Ice Masses'; (1)

The limitations of treating an avalanche as a unified mass are obvious, and this approach will not be discussed further here. It is probably most suitable in the initial stages of motion. Fluid flow approaches are more commonly used. However, they must not rely on inaccessible quantities. Rigorous models which rely on knowledge of such things as temperature profile, free water content, and velocity profiles have been proposed, but remain impractical. Most fluid flow treatments stem from the work of Voellmy (1955), who was the first to regard flowing snow as a fluid and apply analogies results from open-channel flow. Voellmy-type models generally assume that terminal velocity is reached very quickly and ignore mass entrainment and loss in the track.

The velocity V is determined by the component of weight parallel to the slope and a number of resistive forces:

- i) Kinetic friction at the bed surface, which decreases with increasing velocity,
- ii) Viscous shear in the moving snow, proportional to velocity,
- iii) Turbulent resistance against the bed, proportional to the square of velocity,
- iv) Aerodynamic drag at front and top surfaces, proportional to the square of velocity,
- v) Snowcover resistance, independent of velocity .

• Voellmy, A; "On the Destructive Forces of Avalanches";
USDA - Alta Avalanche Study Center; Translation #2, 1964

The resulting equation of motion, with forces indicated below the terms, is

$$m(dV/dt) = mg \sin \beta - [(a_0/V) + a_1 + a_2V + a_3V^2] .$$

(weight) (i) (v) (ii) (iii,iv)

An analytical solution to this equation is not possible if m , β , and a_i are allowed to vary with time. Thus simplifications must be made in order to obtain an approximate analytical expression. It is commonly assumed that the rate of travel is the terminal velocity throughout most of the track, and that m , β , and a_i are constant once terminal velocity is reached. In addition, certain resistive terms are typically neglected.

The equation for terminal velocity V_T is obtained by letting dV/dt equal zero,

$$mg \sin \beta = (a_0/V_T) + a_1 + a_2V_T + a_3V_T^2 .$$

It is now possible to consider the nature of the dependence of terminal velocity V_T on the slope incline β for situations where the resistive force is primarily due to one of the non-constant terms.

If kinetic friction is responsible for most of the resistive force, then

$$V_T = A_1/\sin \beta ; \quad A_1 = a_0/mg = \text{constant} .$$

If viscous shear dominates,

$$V_T = A_2 \sin \beta ; \quad A_2 = mg/a_2 = \text{constant} .$$

If turbulent resistance and aerodynamic drag are the key resistive factors, then

$$V_T = [A_3 \sin \beta]^{\frac{1}{2}} ; \quad A_3 = mg/a_3 = \text{constant} .$$

In practice, it is common to assume that the coefficient of kinetic friction remains constant and that the resistive forces proportional to V (viscous shear) can be neglected. The first assumption allows the (a_0/V) term to be absorbed by the constant a_1 . The second assumption is equivalent to setting a_2 equal to zero. While viscous shear is not understood very well, it appears to be a small effect in comparison with the other terms. The resulting equation for terminal velocity is

$$mg \sin \beta = a_1 + a_3 V_T^2 ,$$

so that

$$V_T = [(mg \sin \beta - a_0)/a_2]^{\frac{1}{2}} .$$

This is usually written in the form[■]

$$V_T = [\tau R(\sin \beta - \mu \cos \beta)]^{\frac{1}{2}}$$

where $a_0 = \mu \cos \beta$ is by analogy to the unified mass equation and $\tau R = 1/a_2$. Although τ is generally referred to as the coefficient of turbulent friction, V_T increases with τ . This is because $\tau = R/a_2$ where a_2 is actually proportional to the V^2 resistive force.

■ for a different approach which also leads to this equation, see Ch. 7 in "Dynamics of Snow and Ice"; (1).

In fast flowing avalanches and powder avalanches, μ is approximately zero. Then we have the simplification

$$V_T = [\tau R \sin \beta]^{\frac{1}{2}} .$$

This is similar to the Chezy equation for the motion of fluids in a channel with the Chezy discharge coefficient $C = \tau^{\frac{1}{2}}$. Speeds calculated from this equation agree well with observed speeds provided the recommended value for τ is used. This depends on terrain.○

Values for μ are 0.2 - 0.3 for speeds between 30 and 50 m/s and 0.1 - 0.15 for speeds between 30 and 50 m/s. For velocities over 50 m/s, μ is usually neglected. Determination of hydraulic radius R must account for slab thickness, surface area of the slab in the starting zone, and track cross section.

Impact pressure is calculated differently for powder and flowing avalanches. In powder avalanches, the impact is analogous to the impingement of a water jet on a fixed surface and the drag force per unit area is P_p is

$$P_p = C \sigma_a V^2 / 2$$

where C = drag coefficient

σ_a = avalanche density

V = avalanche velocity .

The drag coefficient C is dependent on the size and shape of the obstacle. Values may be found in tables of wind pressure on structures, which are published in building codes.

○ see Table 11.2 in "Handbook of Snow"; (11)

When dense, flowing snow encounters an obstacle it is first compacted and then flows around the object. The initial plastic deformation results in an initial peak pressure P_i . When flow around the obstacle is established, pressure drops to the stagnation value P_a :

$$P_a = \sigma_a V^2 .$$

P_i is dependent on the deformation properties of the snow, but appears to be two to three times P_a . Data on this is sparse.

The most common equation used to estimate runout distance D is

$$D = V^2 / \{ 2g [\mu \cos \alpha + \tan \alpha + (V^2 / 2\tau h_m)] \} ,$$

where α = slope in runout zone

μ, τ = coefficients of friction

h_m = average depth in runout zone .

There are numerous problems inherent in the use of this formula. The coefficients of friction in the runout zone are poorly understood, and the definition of where the runout zone begins is somewhat arbitrary. An accurate estimate of D depends heavily on experience and intuition.

Modeling

Since field observations of avalanches are cumbersome, costly, and limited, it is desirable to complement them with laboratory studies. Powder avalanches behave in a newtonian manner, so most models have focussed on this type of avalanche.

When modeling large scale phenomena in a laboratory, it is necessary to maintain geometric and dynamic similarity. Dynamic similarity is ensured by matching the characteristic numbers of the dimensionless equations of motion. It is usually very difficult to match more than two characteristic numbers simultaneously. Thus, models are often restricted in order to eliminate one or more of the numbers. If a moving avalanche is to be modeled using the Navier-Stokes equation and continuity, then the parameters to be matched are the Euler, Reynolds, Froude, and Strouhal numbers. These depend on pressure (P), density (σ), velocity (V), viscosity (μ), and the characteristic length (L) and time (T).[⊙]

The Strouhal number (S) is the ratio of unsteady force to inertial force. If the model is restricted to regions where frontal velocity is nearly steady, then S need not be matched. This limits the model to motion which does not depend on temporal variations, such as vortex shedding.

[⊙] see, for example: Sabersky, Acosta, Hauptmann; "Fluid Flow"; (1971); Sec. 5.2

The Euler number (Eu) is the ratio of pressure to inertial force. This can be neglected since the gradient of atmospheric pressure is very small compared to inertial forces.

The Reynolds number (Re) is the ratio of inertial force to viscous force. As it increases the flow changes in nature from laminar to turbulent. Since avalanches become fully turbulent very quickly, it is sufficient to use a model which has a high enough value of Re to ensure that the flow is turbulent.

Thus the primary requirement in such a model is to match the Froude number (Fr), which is the ratio of inertial forces to gravity forces. Fr for developed avalanche flow is usually greater than unity, so that the flow has a 'shooting' nature. Since Fr exceeds one, the development of hydraulic jump is possible. However, this has not been observed.⊙⊙

The condition which the Froude number imposes is

$$[V^2/\alpha L]_{\text{model}} = [V^2/\alpha L]_{\text{nature}} ,$$

where $\alpha = g \sin \beta - \mu g \cos \beta$ and the characteristic length L is typically taken to be the hydraulic radius. To model a natural avalanche ten meters in height with speeds of 10 - 50 m/s the model velocity would have to be in the range 1 - 5 m/s. This would require inclined open channels ten meters long.

Most modeling which has actually been carried out has used a salt solution wedging under a water ambient.☉ In this type of set-up the ambient acts with a significant buoyant force on the heavier 'avalanche' fluid. The gravity force (αL) in the Froude number must then be replaced by

$$\alpha L[(\sigma_{\text{aval}} - \sigma_{\text{amb}})/\sigma_{\text{aval}}] .$$

The resulting characteristic number to be matched is called the densimetric Froude number and is

$$Fr = (V^2/\alpha L)[(\sigma_{\text{aval}} - \sigma_{\text{amb}})/\sigma_{\text{aval}}] .$$

The density ratio ($\sigma_{\text{aval}}/\sigma_{\text{amb}}$) is on the order of 10 or more in a natural powder avalanche. In the laboratory salt solution model this ratio is between 1.01 and 1.15. Thus a flow which satisfies the Boussinesq approximation (small density difference) is used to model a flow for which this approximation is not valid. An understanding of the inertial effects of large density variations is needed to properly interpret the results of these models. It is suspected that density effects will result in a decreased growth rate for the avalanche and will change the amount of aerodynamic drag. The internal structure and the dynamics should remain similar to the Boussinesq approximated model.☉♥

☉ "A Model Study of Powder-Snow Avalanches"; (37)

☉♥ Ibid.

VI - Wave Propagation and Detonations in Snow

The propagation of stress waves through the snowpack is a problem of great interest and one with numerous applications. Despite this, much less work has been done on this topic than on others. Much of the work which has been done has been experimental in nature. Thus there are many open problems, especially related to the development of a satisfactory basic theory.

Explosives are used often in snow. Most ski areas and transportation corridors prone to avalanches are managed through the use of explosives. Explosives are used both for control and for stability analysis. They can be used to relieve extra stresses and reduce hazards by causing avalanches to occur at a time when minimal damage will occur and by preventing the buildup of stresses which could result in much larger avalanches.

Propagation of plastic waves is also an important factor in the operation of railroads on snow covered track and in the fast removal of snow from roads. Under certain conditions the snow can exert a particularly large resistive force against the vehicle. These conditions can be better understood through an analysis of stress wave propagation.

A - Use of Explosives

The primary use of explosives in the snowpack is to release avalanches in a controlled manner. Every year in Switzerland somewhere between ten and fifteen thousand charges are used. Avalanches result 20 to 30 percent of the time.⊙ The objective is to destroy snowpack stability through increased stress and/or decreased strength. Explosions create stress waves which can travel through the snow, the air above, and the ground below. When the explosion occurs within the snowpack, a bubble of rapidly expanding gas can act to pressurize cracks and pores in the snow.

There are three means of presenting the charge which can be used. The two most common are delivery by hand and projectile delivery. The first method involves an individual, such as a ski patroller, tossing a charge from a ridge line or other stable area down into a starting zone. A variation is the use of a "trolley", a wire strung downslope using trees or posts. The second method makes use of artillery type hardware. The most common are the 75mm recoilless, 75mm howitzer, and 105mm recoilless. Also in use is the 'avalauncher', a light and mobile system which was designed for this purpose. Highway control typically relies on projectile delivery while ski area control usually involves both methods.

The third method is that of preplacement, in which charges are placed before the development of the snowpack and set off when

⊙ "Artificial Release of Avalanches by Explosives"; (34)

needed. Clearly, caution must be taken in any area with public access. This method is rarely, if ever, used in the U.S. but is in use in certain areas of Europe.

There are three types of blasts which can be used, blasts in the snow, in the air, and on the underlying ground. Blasts in the snow can cut the snow normal to the fall line and also apply downslope thrust. They can be particularly useful when dealing with coherent slabs. Surface blasts apply a brief (10 msec) increase in normal and downslope shear stresses. The overpressure required to release an avalanche varies widely, but a lower limit is indicated by sonic booms from aircraft, which occasionally release slides with an approximate overpressure of 2 lbf/ft². In dry snow an airblast of 1 kg detonated 1-2 meters above the snow has been found to have an effective range between 17 and 120 meters, whereas a similar blast in the snow had a range under 6 meters.● Widespread simultaneous air shots are more effective than single ones. In wet snow, however, it may be necessary to place the charge in the snow.

A ground placement can often transmit shock waves further since rock has less attenuation than snow. However, this method does not seem to have found widespread use. The best placement and method of delivery will vary with conditions and are often chosen based on the structure of the snowpack known or suspected to exist, as well as availability of resources and accessibility of the target.

● Ibid.

B - Effect of Explosives on Snow

An explosion can be defined as the rapid generation of energy in a limited space. Energy is typically produced at a rate exceeding the local dissipation rate of the surrounding medium. Usually a violent gas expansion is involved and very high pressures develop rapidly. This gas expansion provides most of the energy associated with an explosion. If pressure and stress waves created by the explosion travel at speeds in excess of the acoustic velocity of the unreacted medium, the explosion is called a detonation. Otherwise it is called a deflagration.

There are several types of waves which travel outwards from an explosion. An elastic wave is one which travels with no internal dissipation in the medium, so the medium is in the same state before and after the wave passes. A plastic wave is one which causes yielding and viscous flow of the medium. If a parameter is discontinuous across a wave front, the difference between its value just ahead of the wavefront and its value just behind the wavefront is called its jump. If particle displacement is continuous across the wavefront but particle velocity has a jump, the wave is called a shock wave. A wave is called steady if it propagates through the medium at a constant velocity. In a steady shock wave, the wave moves through the medium at a constant velocity but an individual particle within the medium will change velocity abruptly as the wavefront passes.

Effective "coupling" allows efficient transmission of energy away

from the point where an explosion occurs. Good coupling requires physical contact between the explosive and the medium and "impedance matching", or matching the product of detonation velocity and explosive density to the product of the acoustic velocity and the density of the medium.♥ Snow has low strength and density and is porous. It is very energy-absorbing and attenuates waves rapidly. Coupling is typically very poor when explosives are used in snow. Hugoniot curves for snow show that snow with a density of .4 g/cm³ is compressed about 50% by pressures of approximately 20 bars.♦ Rapid attenuation due to these losses to the medium and to geometrical spreading result in elastic waves travelling at or below acoustic velocity except for a region very close to the point of explosion. While coupling is poor in snow, it is good (close to unity) in frozen soil. As a result, shock waves can travel much further in the ground under the snow if the explosives are set off on the ground.

It is standard procedure in work related to explosives to scale quantities in order to remove the effect of charge size. Linear dimensions are usually scaled with respect to the cube root of charge weight, resulting in units such as ft/(lb)^{1/3}. In general,

$$d_2/d_1 = t_2/t_1 = I_2/I_1 = (W_2/W_1)^{1/3}$$

where

-
- ♥ Parameters such as detonation velocity and explosive density can be found in resources such as DuPonts' "Blasters Handbook"
 - ♦ "Dynamic Response of Snow", (9), is a classic source of Hugoniot curves for snow.

d = distance (crater size, specified isobar, etc.)

t = time (arrival time, phase duration, etc.)

I = explosive impulse

W = weight of equivalent TNT charge (1 ton = 10^9 cal.) .

Near the point where an explosion occurs a crater is formed when the snow is fragmented, compacted, fused and vaporized. Most of the energy is absorbed in this crater region. The 'crater' actually consists of four zones:

- 1) The apparent, or immediately visible, crater ,
- 2) The true crater, or zone of total fragmentation,
- 3) The "(complete) rupture" zone, which includes the fragmented and deformed area immediately around the true crater, and
- 4) The plastic, or "extreme rupture" zone, which contains the most distant snow permanently deformed.

Typical apparent crater dimensions are $2.8 \text{ ft}/(\text{lb})^{1/3}$ radially and $1.8 \text{ ft}/(\text{lb})^{1/3}$ in depth for surface shots and $3.2 \text{ ft}/(\text{lb})^{1/3}$ radially and $2.0 \text{ ft}/(\text{lb})^{1/3}$ in depth for subsurface shots. The maximum dimensions occur when the charge depth is $1.3 \text{ ft}/(\text{lb})^{1/3}$.♣

While intense adiabatic compression occurs near the explosion, tangential stresses (tensile hoop stress) causing radial cracking occur further away. At free boundaries with rock or air tensile waves reflect and surface spalling may occur.

♣ "Avalanches", (7) and "Controlled Release ... ", (8)

Mechanical fracturing around the crater can branch into weak regions and release slabs. The importance of brittle fracture is indicated by the general ineffectiveness of explosives in wet slabs.

Other effects include thrust and the generation of stress and pressure waves. Thrust is maximized by good coupling, but appears to be of little importance except in cornice blasting. Stress and pressure waves travel poorly in snow but much better in air. In the underlying ground the most important stress waves are those which travel along the surface and have the largest amplitude. These are called Rayleigh waves.

VII - Glossary

- Albedo - fraction of incident radiation reflected
- Avalanche - a large mass of snow, possibly mixed with rocks, soil and ice, moving rapidly down an incline
- Cornice - hard deposit of wind-drifted snow sometimes found on the lee edge of a ridge, it is often overhanging
- Dynamic Adiabatic - see Hugoniot
- Elastic - elastic deformation is entirely recoverable; an elastic material 'remembers' its original state
- Euler number - fluid flow similarity parameter, the ratio of pressure to inertial force
- Front - a zone separating large masses of air having significantly different physical properties (particularly temperature and moisture content)
- Froude number - fluid flow similarity parameter, ratio of inertial force to body force
- Grain - a mechanically separate particle in the snowcover; may or may not be a single crystal
- Graupel - a form of precipitated snow, generally rounded ('snow pellets'), which results when crystals fall through a region of cloud where extreme riming occurs
- Habit - crystal orientation (e.g. plate-like, prism-like)
- Hugoniot - the locus of states attained by a solid under the effect of shock compression

Ice Crystal - initial stage in the growth of a snow particle, having a small size ($\approx 7.5 \mu\text{m}$ or less), a low fall velocity ($\approx 5 \text{ cm/s}$ or less), and a very simple crystal form (often a hexagonal plate)

Needle - exaggerated form of hollow prism in which both the prism and the basal faces are often incomplete

Reynolds number - fluid flow similarity parameter, ratio of inertial force to viscous force; a measure of the turbulence present

Rime - created when small ($10\text{-}40 \mu\text{m}$) cloud droplets freeze on contact onto an exposed surface such as a snowflake, or vegetation

Sastrugi - an erosional surface form of dry snow subject to scouring by wind, consists of wavelike forms with sharp prows towards the prevailing winds

Sector Plate - a plate-like crystal in which six-fold branching occurs along the a axes

Snow Crystal - a single crystal with a common orientation of the molecules within it; it may or may not exhibit external evidence of this order

Snowflake - an aggregation of snow crystals

Strouhal number - fluid flow similarity parameter, ratio of unsteady force to inertial force

Surface Hoar - plate-type ice crystals which form by deposition of water vapor onto the snow surface during the night

Viscoelastic - a viscoelastic material behaves in a way which is partly elastic and partly viscous; deformation is partially recoverable

Viscous - deformation (or 'flow') which is entirely unrecoverable; a viscous material has no 'memory' of an initial, or reference state

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